# 06 FEB 12

•Day 89•

- HW ✓ & Q:A with WARM-UPS
- Physics : a Diff Eq
- Introduction to Momentum (HRW Ch. 9)
  - → collision demo
  - → intro to $p = mv$
  - → center of mass
- Midterm Examination
  - → description : rationale of exam format
  - → review
- Homework Assignment → STUDY

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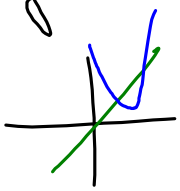
# Warm-ups



① What is the height of a triangle of base length 2 such that it contains the same area as the region bounded by $y = x$ and $y = x^2 - 3x + 3$ ?



$x^2 - 3x + 3 = x$

$x^2 - 4x + 3 = 0$

$(x - 3)(x - 1) = 0$

$$\int_1^3 \left[x - (x^2 - 3x + 3)\right] dx = 4/3$$

$\frac{1}{2} b h = 4/3$

~~$\frac{1}{2}(2) h = 4/3$~~

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② Volume given by $V = \left(\frac{\pi}{8} x^2 y^3\right)$

$\frac{dx}{dt} = \frac{dy}{dt} = 3$

find $\frac{dV}{dt}$ when $x = 1$ and $y = 2$

$$\frac{dV}{dt} = \left(\frac{\pi}{8} x^2\right)\left(3y^2 \frac{dy}{dt}\right) + \left(y^3\right)\left(\frac{\pi}{4} x \frac{dx}{dt}\right)$$

# Warm-up

(3) $f(x) = \begin{cases} 3x^2 - 2 & , \ x \geq 1 \\ -2x + 3 & , \ x < 1 \end{cases}$

What can you say about $f(x)$ at $x = 1$?

$3(1)^2 - 2 = 1$

$-2(1) + 3 = 1$

} continuous @ $x = 1$

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$6x$

$-2$

} @ $x = 1$ $\quad 6(1) = 6$, $\quad -2 = -2$

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## ⇒ Physics

solving the Diff Eq

$a(t) = 3t$

$v(1) = 2$

$x(1) = -1$

} find $x$ at $t = 2$

$$a = \frac{dv}{dt} \Rightarrow \frac{dv}{dt} = 3t \Rightarrow \int dv = \int 3t\,dt$$

$$v = \frac{3t^2}{2} + C$$

$$2 = \frac{3(1)^2}{2} + C$$

$$2 = \frac{3}{2} + C$$

$$C = \frac{1}{2}$$

$$v(t) = \frac{3t^2}{2} + \frac{1}{2}$$

$$v = \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{3t^2}{2} + \frac{1}{2}$$

$$\int dx = \int \left(\frac{3t^2}{2} + \frac{1}{2}\right) dt$$

$$x = \frac{t^3}{2} + \frac{1}{2}t + C$$

$$-1 = \frac{(1)^3}{2} + \frac{1}{2}(1) + C$$

$$-1 = \frac{1}{2} + \frac{1}{2} + C \Rightarrow C = -2$$

$$\boxed{x(t) = \frac{t^3}{2} + \frac{1}{2}t - 2}$$

$$x(2) = 4 + 1 - 2$$

$$\boxed{x(2) = 3}$$

# ⇒ Newton's Original 2nd Law of Motion

- $\vec{p} = m\vec{v}$ $\quad p \equiv$ momentum

  momentum is a vector quantity

  "p=mv" notation was developed after Newton although he developed the associated ideas

- Newton's concept: "momentum" is the tendency of a moving mass to continue moving in the absence of a non-zero net force.

- "Impetus" was probably used before "momentum"

"Impetus" is from the Latin "petere" which means "to go towards" or "rush upon" or "to seek" thus, from "petere" we have the "p"

- UNITS: [m] is kg, [v] is m/s } [p] is ∴ kg·m/s

- $\Delta\vec{p} = m\Delta\vec{v}$ (assumes $\Delta m$ is zero)

  $= m(\vec{v}_f - \vec{v}_i)$

  in one dimension: $\Delta p_x = m(v_{xf} - v_{xi})$

⇒ Why would the velocity ever change?

- However, Newton did not directly state for his second law that "$F = ma$" or "$a = F/m$"

Instead...

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$$F = ma \quad \text{or} \quad a = \frac{F}{m}$$

$$F = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt}$$

A person holds a portable fire extinguisher that ejects 1.0 kg of water per second horizontally at a speed of 6.0 m/s. What horizontal force in newtons must the person exert on the extinguisher in order to prevent it from accelerating?

(A) 0 N
(B) 6 N
(C) 10 N
(D) 18 N
(E) 36 N

$$F = \frac{d(mv)}{dt} = \cancel{m\frac{dv}{dt}}^{0} + v\frac{dm}{dt}$$

$$= \left(6\ \tfrac{m}{s}\right)\left(1\ \tfrac{kg}{s}\right) = 6N$$

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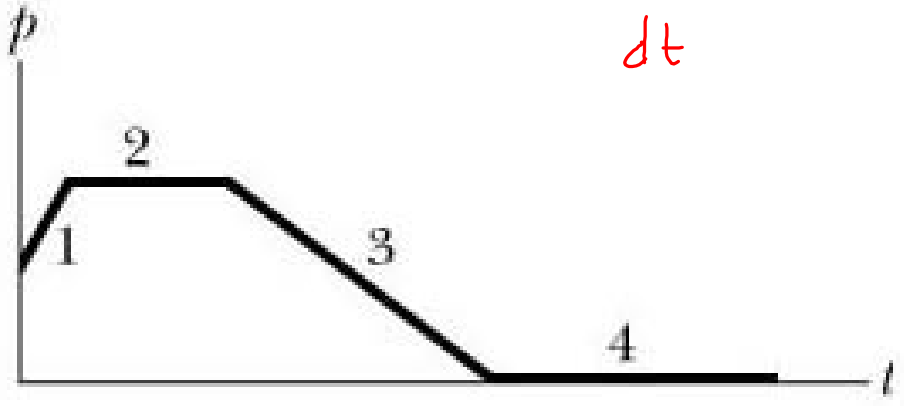
$p = mv$

$\vec{F} = \frac{d\vec{p}}{dt}$




(a) rank the four regions according to magnitude of force from greatest to least.

(b) in which region is the particle slowing?

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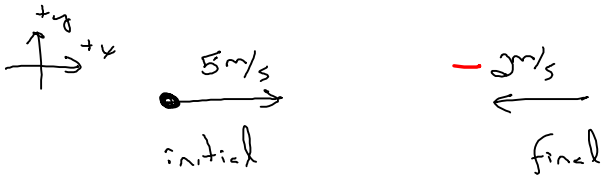
•18 A 0.70 kg ball moving horizontally at 5.0 m/s strikes a vertical wall and rebounds with speed 2.0 m/s. What is the magnitude of the change in its linear momentum?




final − initial

$\Delta p = m\Delta v = m(v_f - v_i)$

$= 0.7\,kg\,(-2\,m/s - +5\,m/s)$

$= 0.7\,kg\,(-7\,m/s)$

$= -4.9\,kg\cdot m/s$

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- Calculus: 25 MC @ 1 pt each
  FRQ's 25 pts total

- Physics: 24 MC @ 1.5 pts each
  FRQ's 36 pts total

- MC → chose problems from book software
  similar to AP if not more difficult
  so, chose less complex/simpler AP style MC
  conceptual & quantitative

- Most FRQ's have been done in class
  or for homework
  must show all work for full credit

- many Calculus MC have been homework,
  on tests/quizzes, practice WS., or warm-ups

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$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f(x) = \ln(x) \Rightarrow f'(4)$ , $x > 0$

$$\lim_{h \to 0} \frac{\ln(x+h) - \ln(x)}{h}$$

$$\lim_{h \to 0} \frac{\ln(4+h) - \ln(4)}{h} = \frac{1}{4}$$

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$$\lim_{h \to 0} \frac{\sin\left(\frac{\pi}{6} + h\right) - \frac{1}{2}}{h}$$

$f(x) = \sin x$

$f'(x) = \cos x$

$f'\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

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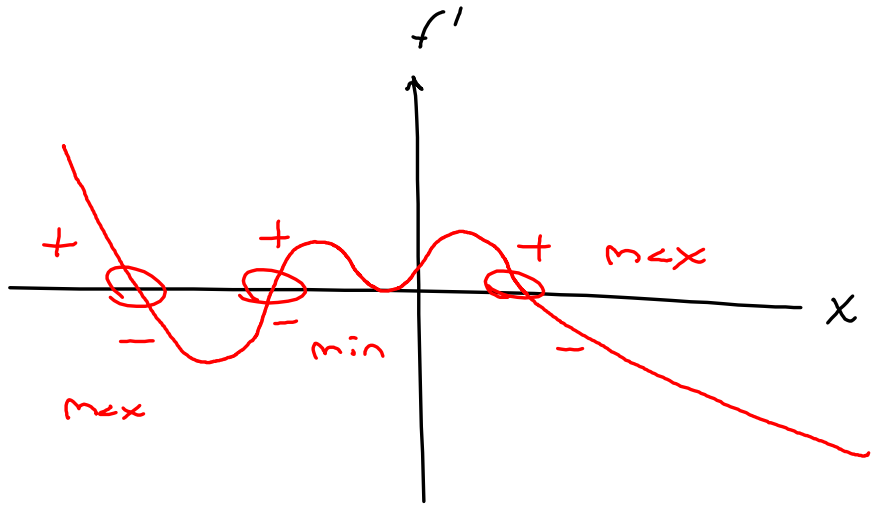


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$$\sin x + \cos y + \sin(xy) = 3$$

$$\cos x - \sin y \frac{dy}{dx} + \cos(xy)\left(x\frac{dy}{dx} + y\right) = 0$$

$$\cos x - \sin y \frac{dy}{dx} + x\cos(xy)\frac{dy}{dx} + y\cos(xy) = 0$$

$$-\sin y \frac{dy}{dx} + x\cos(xy)\frac{dy}{dx} = -\cos x - y\cos(xy)$$

$$\left[-\sin y + x\cos(xy)\right]\frac{dy}{dx} = -\cos x - y\cos(xy)$$

$$\frac{dy}{dx} = \frac{-\cos x - y\cos(xy)}{-\sin y + x\cos(xy)}$$

$f'(x) = 3x + 1$ & $f(2) = -3$

approximate $f(1.9)$

$f'(2) = 3(2) + 1 = 7$

$y - y_1 = m(x - x_1)$

$y - -3 = 7(x - 2)$

$y + 3 = 7x - 14$

$y = 7x - 17$

$y = 7(1.9) - 17 = -3.7$

$\frac{dy}{dx} = 3x + 1$

$y = \frac{3x^2}{2} + x + C$

$$-3 = \frac{3(2)^2}{2} + 2 + C$$

$$-3 = 6 + 2 + C \Rightarrow C = -11$$

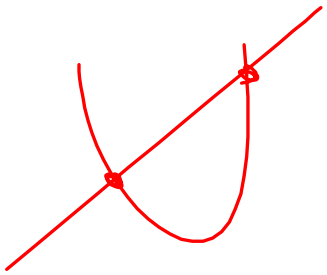
$$y = \frac{3x^2}{2} + x - 11$$

$$-3.685$$

Monday, February 06, 2012
9:11 AM

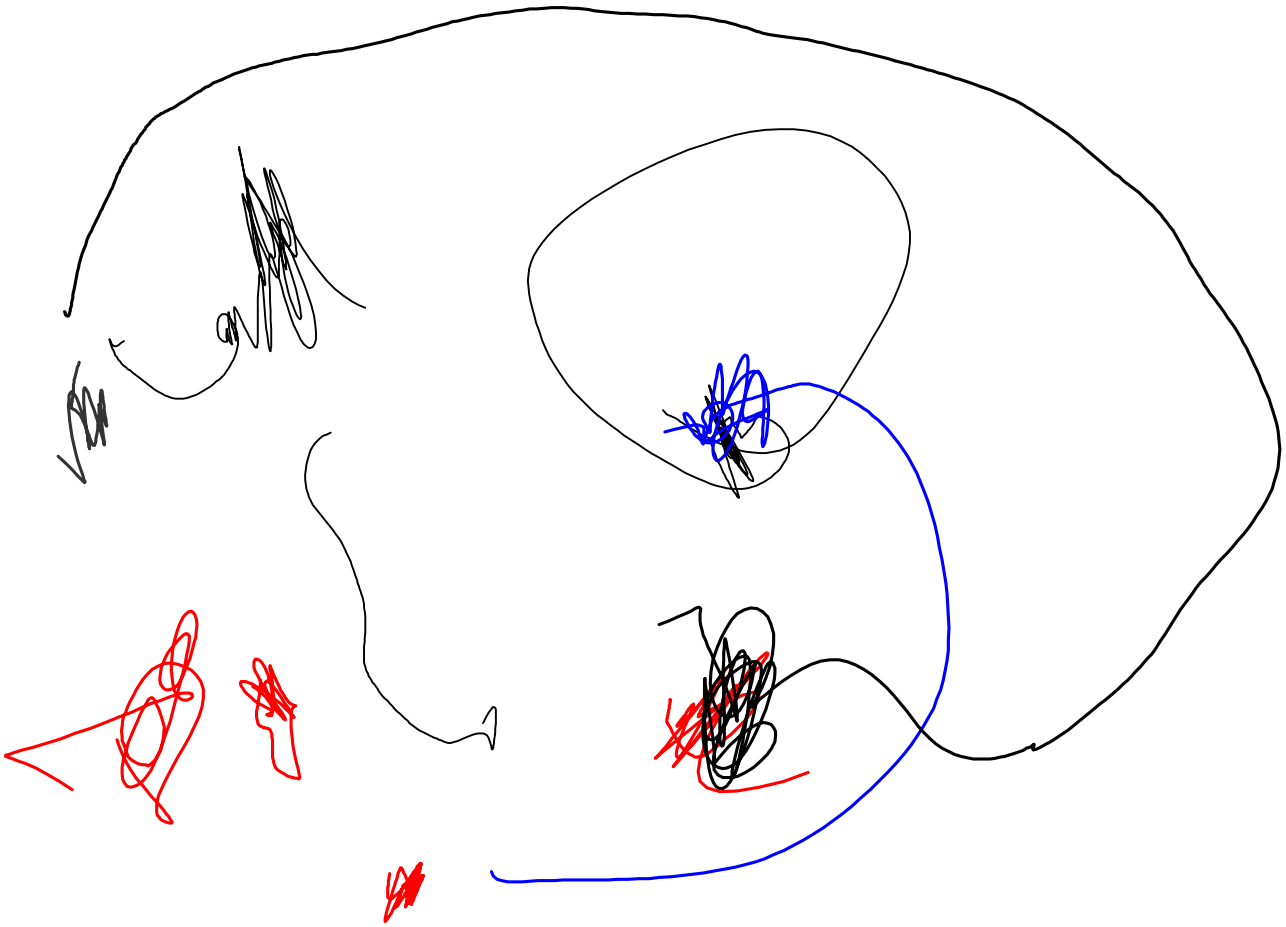
bottom $Y_1 = x^2 - 3x + 3$

top $Y_2 = X$



$$fnInt(Y_2 - Y_1, X, 1, 3) = \frac{4}{3}$$

$$fnInt(X - (x^2 - 3x + 3), X, 1, 3)$$



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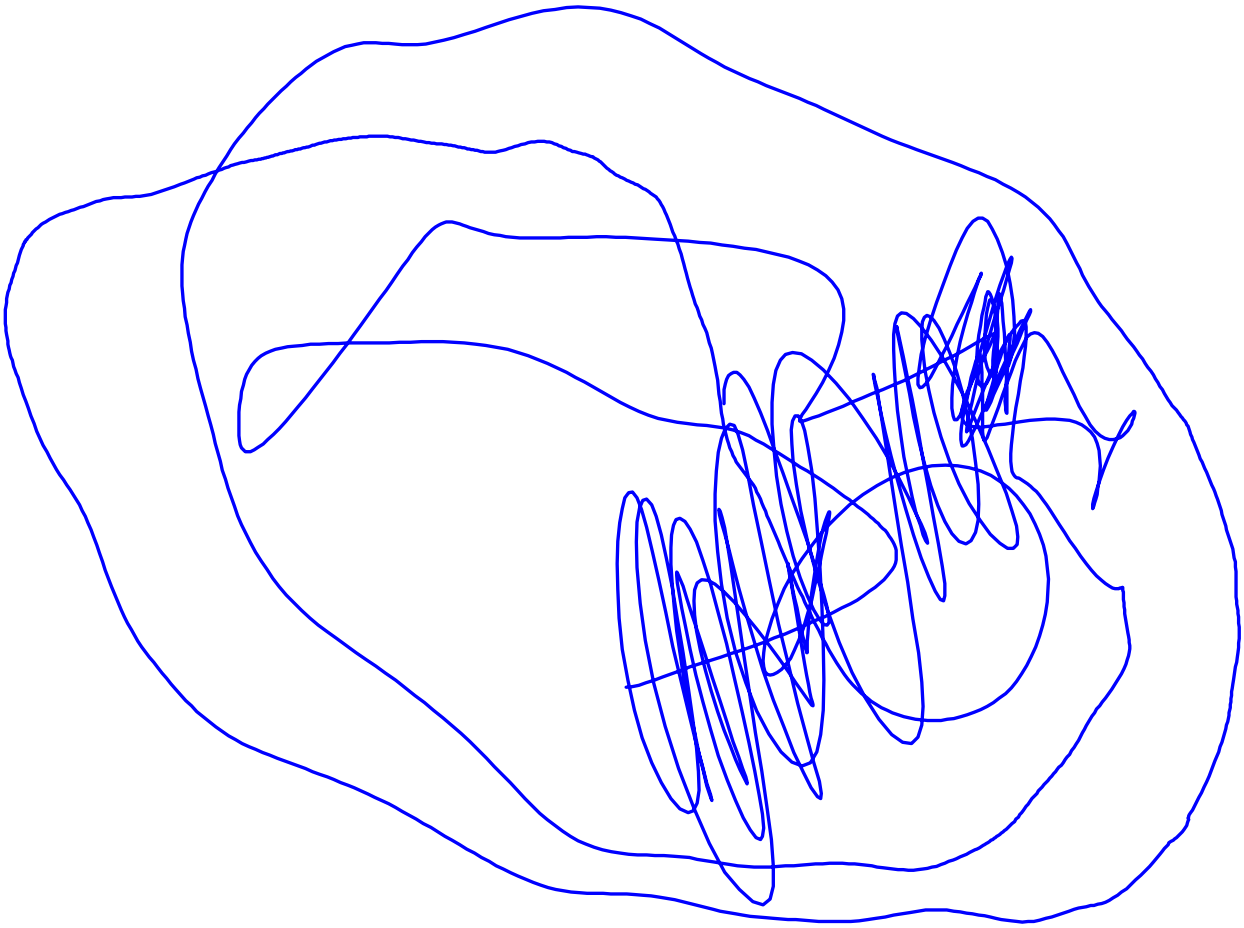

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# ★Homework Assignment★

- study for Midterm Examinations

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