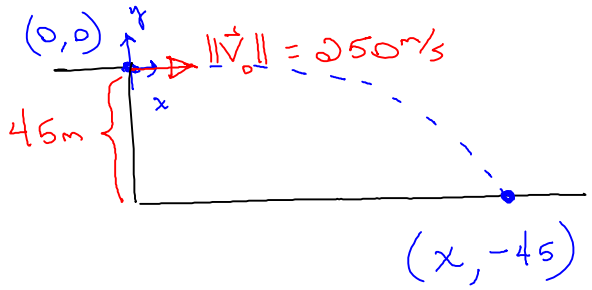
08 JAN 10

★ turn-in your related rates projects ★

- HW Q & A
- Projectile Motion cases
- scan & midterm questions
- homework assignment

We'll have a study session this Sunday from 1pm → 4pm

•21 A projectile is fired horizontally from a gun that is 45.0 m above flat ground, emerging from the gun with a speed of 250 m/s. (a) How long does the projectile remain in the air? (b) At what horizontal distance from the firing point does it strike the ground? (c) What is the magnitude of the vertical component of its velocity as it strikes the ground?



$v_x = v_0 \cos\theta = 250 \cos 0^\circ = 250 m/s$

$v_{0y} = v_0 \sin\theta = 250 \sin 0^\circ = 0$

(a) $y = \cancel{v_{0y} t}^{0} + \frac{1}{2} a t^2 \Rightarrow -45 = \frac{1}{2}(-9.8)t^2 \Rightarrow t = \left(\frac{-45}{-4.9}\right)^{1/2}$

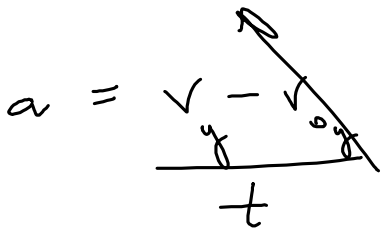
$t = 3.030 s$

(b) $v_x = \frac{x}{t} \Rightarrow x = v_x t = (250)(3.030) = 757.5 m$

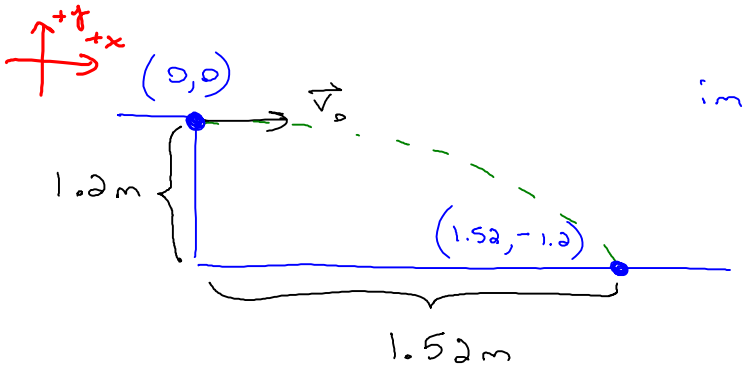
(c) $y = \frac{v_y^2 - \cancel{v_{0x}^2}^{0}}{2a} \Rightarrow -45 = \frac{v_y^2}{2(-9.8)}$

$v_y = \pm 29.698 m/s$

$\|\vec{v}_y\| = 29.698 m/s$



•24 A small ball rolls horizontally off the edge of a tabletop that is 1.20 m high. It strikes the floor at a point 1.52 m horizontally from the table edge. (a) How long is the ball in the air? (b) What is its speed at the instant it leaves the table?



implied that $v_{y_0} = 0$

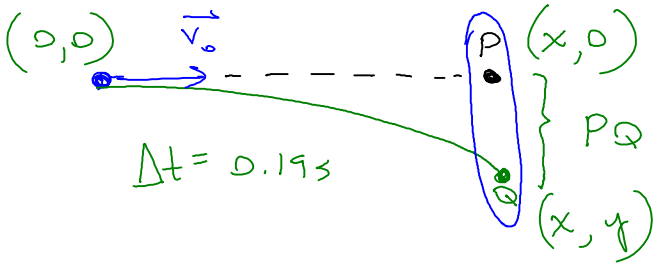
$$y = \cancel{v_{0y}^{0}} t + \tfrac{1}{2} a t^2$$

$$-1.2 = \tfrac{1}{2}(-9.8)\, t^2$$

$$t = \left(\frac{-1.2}{-4.9}\right)^{1/2} = 0.495\,s$$

$$v_x = \frac{x}{t} = \frac{1.52}{0.495} = 3.071\ m/s$$

•25 A dart is thrown horizontally with an initial speed of 10 m/s toward point P, the bull's-eye on a dart board. It hits at point Q on the rim, vertically below P, 0.19 s later. (a) What is the distance PQ? (b) How far away from the dart board is the dart released?



implied $v_{0y} = 0$

$$y = \overset{0}{v_{0y}}\,t + \tfrac{1}{2}at^2$$

$$PQ = \tfrac{1}{2}(-9.8)(0.19)^2$$

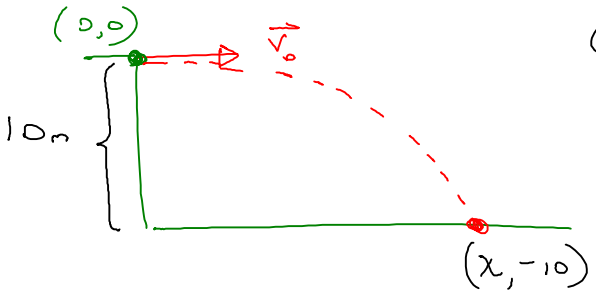
$$PQ = \boxed{0.177\text{m}}$$

$$v_x = \frac{x}{t} \Rightarrow x = v_x t = (10)(0.19) = \boxed{1.9\text{m}}$$

••29 A lowly high diver pushes off horizontally with a speed of 2.00 m/s from the platform edge 10.0 m above the surface of the water. (a) At what horizontal distance from the edge is the diver 0.800 s after pushing off? (b) At what vertical distance above the surface of the water is the diver just then? (c) At what horizontal distance from the edge does the diver strike the water? SSM WWW

implies $v_{0y} = 0$

$\|\vec{v}_0\| = v_x = 2\ m/s$



(a) $v_x = \frac{x}{t} \Rightarrow x = v_x t$

$= (2)(0.8)$

$= \boxed{1.6\ m}$

(b) $y = \cancel{v_{0y} t}^{\,0} + \frac{1}{2} a t^2$

$= \frac{1}{2}(-9.8)(0.8)^2$

$= \boxed{-3.136\ m}$ ★★★

(c) $y = \cancel{v_{0y} t}^{\,0} + \frac{1}{2} a t^2 \Rightarrow t = \left(\frac{2y}{a}\right)^{1/2} = \left(\frac{2 \cdot -10}{-9.8}\right)^{1/2} = \boxed{1.429\ s}$

$v_x = \frac{x}{t} \Rightarrow x = v_x t = \boxed{2.858\ m}$

★★★ but, it asks for distance above the water: $10\ m - 3.136\ m = \boxed{6.864\ m}$

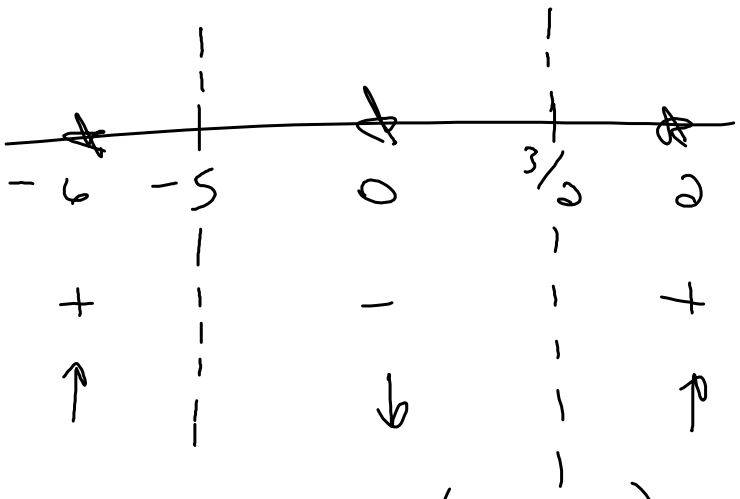
#5 $f(x) = \frac{2}{3}x^3 + \frac{7}{2}x^2 - 15x + \pi$

when $x = 0$, $y = \pi \Rightarrow$ y-int @ $(0, \pi)$

x-ints: $x_1 = -8.100$ $(-8.100, 0)$
$x_2 = 0.221$ $(0.221, 0)$
$x_3 = 2.628$ $(2.628, 0)$

$f'(x) = 2x^2 + 7x - 15$
$(2x - 3)(x + 5)$

$f'(x) = 0$ when $(2x-3)(x+5) = 0$
or $x = \frac{3}{2}$, $x = -5$



$f'(x)$	+	−		+
$f(x)$	↑	↓		↑

↑ on $(-\infty, -5)$, $(\frac{3}{2}, \infty)$
↓ on $(-5, \frac{3}{2})$

by virtue of the First Derivative Test, the point $(-5, f(-5))$ is a relative maximum because $f'(x)$ changes sign from positive

to negative about the value $x = -5$.

$$f''(x) = 4x + 7$$

$$4x + 7 = 0 \Rightarrow 4x = -7 \Rightarrow x = -\frac{7}{4}$$

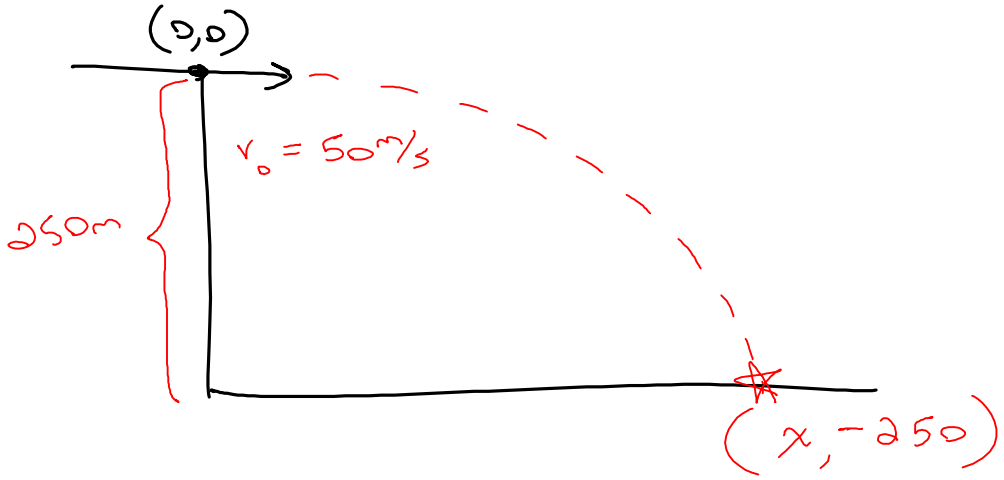
	-2	$-7/4$	0
$f''(x)$	$-$		$+$
$f(x)$	CD		CU

concave down on $\left(-\infty, -7/4\right)$

concave up on $\left(-7/4, \infty\right)$

$\left(-7/4 \;\; f(-7/4)\right)$ is a point of inflection

CASE I



① $v_x = 50 m/s$ and $v_{0y} = 0 m/s$

② find the time of flight

$y = v_{0y} t + \frac{1}{2} a t^2 \Rightarrow -250 = \frac{1}{2}(-9.8) t^2$

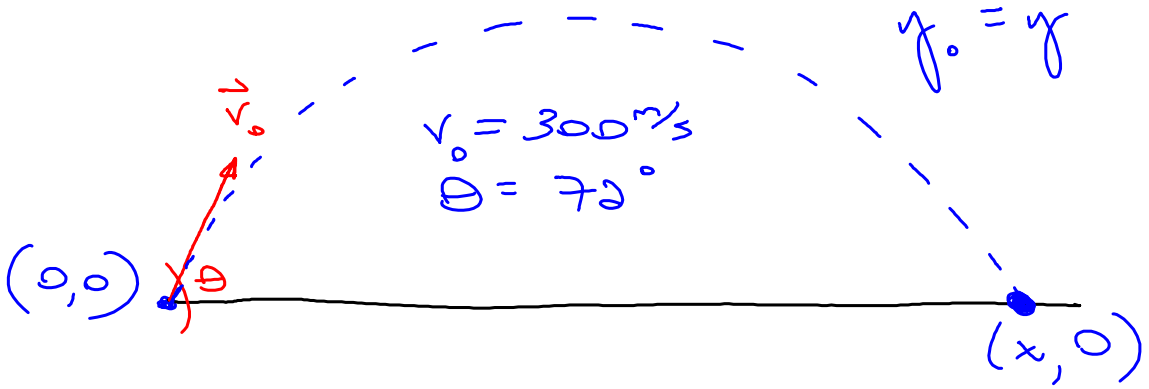
$t = 7.143 s$

③ find the range

$v_x = \frac{x}{t} \Rightarrow x = v_x t = (50)(7.143)$

$= 357.15 m$

CASE II: projectile fired on level ground



(1) $v_x = v_0 \cos\theta = 300\cos 72° = 92.705 m/s$

$v_{0y} = v_0 \sin\theta = 300 \sin 72° = 285.317 m/s$

(2) $y = v_{0y} t + \frac{1}{2} a t^2$

$0 = (285.317)t - 4.9t^2$

$0 = (285.317 - 4.9t)(t)$

$285.317 - 4.9t = 0$, $\boxed{t = 58.228 s}$

(3) $v_x = \frac{x}{t} \Rightarrow x = v_x t = (92.705)(58.228)$

$= \boxed{5398.027 m}$

(4) find the time of y_{max}

$\frac{t_{total}}{2} = \frac{58.288}{2} = 29.144 s$

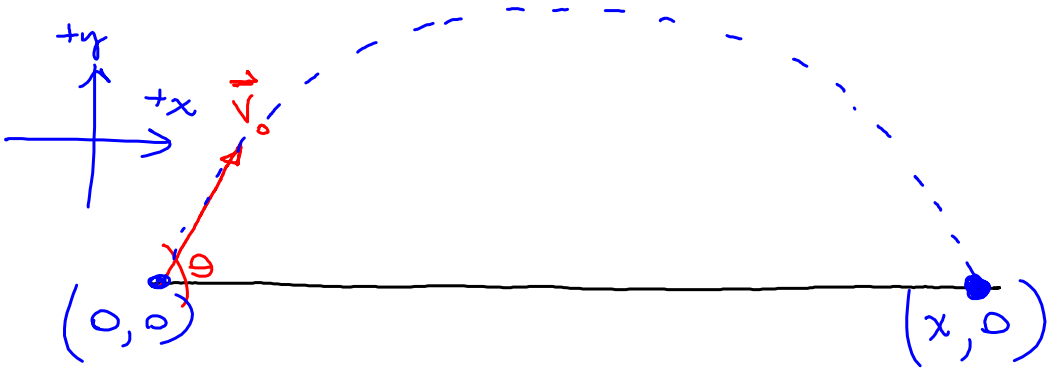
then, find y_{max}. $y = v_{oy}t + \frac{1}{2}at^2$

$y = (285.317)(29.144) + \frac{1}{2}(-9.8)(29.144)^2$

$= 4153.352$ m

CASE II: fired at an angle on level ground

$y_{initial} = y_{final}$ or $y_0 = y = 0$



given: $\theta = 65°$ and $\|\vec{v}_0\| = 89\ m/s$

(1) $v_x = v_0 \cos\theta = 89\cos 65° = +37.613\ m/s$

$v_{0y} = v_0 \sin\theta = 89\sin 65° = +80.661\ m/s$

(2) $y = v_{0y}t + \frac{1}{2}at^2$

$0 = (80.661)t - 4.9t^2$

$0 = (80.661 - 4.9t)t$

$80.661 - 4.9t = 0 \Rightarrow t = 16.461\ s$

(3) $v_x = \frac{x}{t} \Rightarrow x = v_x t = (37.613)(16.461)$

$= 619.148\ m$

(4) exploiting the symmetry

$\frac{t}{2}$ yields the time when the projectile reaches y_{max}

so $t = 8.231\ s$

what is y_{max}? $y = v_{0y}t + \frac{1}{2}at^2$

$= (80.661)(8.231) - 4.9(8.231)^2$

$= \boxed{331.949\ m}$

This only works when $y_0 = y$

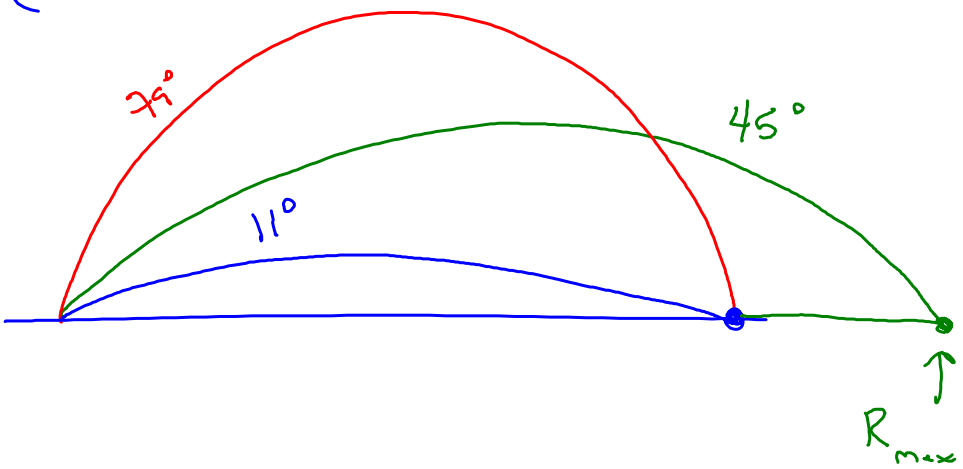
range equation: $R = \frac{v_0^2 \sin(2\theta)}{|g|}$

$= \frac{(89)^2 \sin(2 \cdot 65)}{9.8}$

$= 619.167\ m$

$\sin(2 \cdot 30^\circ) = 0.866$

$\sin(2 \cdot 60^\circ) = 0.866$



Homework

HRW chapter 4 problems

#'s 13, 22, 28, 34, 39, 51

11, 18, 24, 28, 31, 39

7th edition (old book