

11 JAN 10

- HW Q & A
- Related Rates project presentations
- projectile motion case III & case IV
- case V (if time permits)
- TI calculator program xfer
- Homework Assignment

11 JAN 10

VOTE! WHO DREW
THE MOST ACCURATE
ELMO?


WHO DREW THE BEST?



by
Johanna



- HW $Q \neq A$
- Related Rates project presentations
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NEWTON'S METHOD
and a few APPS
- Homework Assignment

•13 The position \vec{r} of a particle moving in an xy plane is given by $\vec{r} = (2.00t^3 - 5.00t)\hat{i} + (6.00 - 7.00t^4)\hat{j}$, with \vec{r} in meters and t in seconds. In unit-vector notation, calculate (a) \vec{r} , (b) \vec{v} , and (c) \vec{a} for $t = 2.00$ s. (d) What is the angle between the positive direction of the x axis and a line tangent to the particle's path at $t = 2.00$ s? 

$$(a) \vec{r}(2) = (2 \cdot 2^3 - 5 \cdot 2)\hat{i} + (6 - 7 \cdot 2^4)\hat{j} = \boxed{6\hat{i} - 106\hat{j}} \text{ in meters}$$

$$\frac{d\vec{r}}{dt} = \vec{v} = (6t^2 - 5)\hat{i} - 28t^3\hat{j}$$

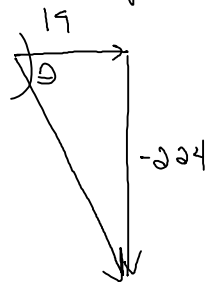
$$(b) \vec{v}(2) = \boxed{19\hat{i} - 224\hat{j}} \text{ in meters per second}$$

$$\frac{d\vec{v}}{dt} = \vec{a} = 12t\hat{i} - 84t^2\hat{j}$$

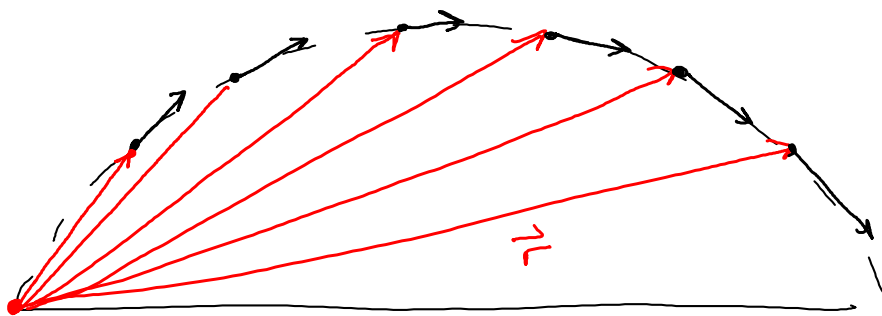
$$(c) \vec{a}(2) = \boxed{24\hat{i} - 336\hat{j}} \text{ in meters per seconds squared}$$

(d) velocity is tangent to the parabolic trajectory of the particle

at 2s \Rightarrow



$$\theta = \tan^{-1} \left| \frac{-224}{19} \right| = 85.152^\circ \text{ below } +\hat{x}$$



•22 In the 1991 World Track and Field Championships in Tokyo, Mike Powell jumped 8.95 m, breaking by a full 5 cm the 23-year long-jump record set by Bob Beamon. Assume that Powell's speed on takeoff was 9.5 m/s (about equal to that of a sprinter) and that $g = 9.80 \text{ m/s}^2$ in Tokyo. How much less was Powell's range than the maximum possible range for a particle launched at the same speed?

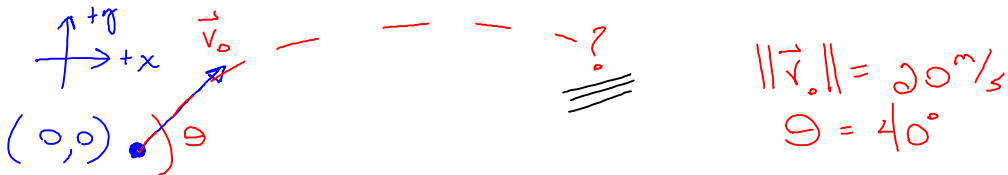
$$\|\vec{v}_0\| = 9.5 \text{ m/s}$$

maximum range when $y_0 = y$ is $\Theta = 45^\circ$ (special case)

$$R = \frac{v_0^2 \sin 2\Theta}{|g|} = \frac{(9.5)^2 \sin(2 \cdot 45^\circ)}{9.8} = 9.209 \text{ m}$$

$$9.209 \text{ m} - 8.95 \text{ m} = \boxed{0.259 \text{ m}}$$

•28 A stone is catapulted at time $t = 0$, with an initial velocity of magnitude 20.0 m/s and at an angle of 40.0° above the horizontal. What are the magnitudes of the (a) horizontal and (b) vertical components of its displacement from the catapult site at $t = 1.10 \text{ s}$? Repeat for the (c) horizontal and (d) vertical components at $t = 1.80 \text{ s}$, and for the (e) horizontal and (f) vertical components at $t = 5.00 \text{ s}$.



① $v_x = v_0 \cos \Theta = 20 \cos 40^\circ = 15.321 \text{ m/s}$
 $v_{0y} = v_0 \sin \Theta = 20 \sin 40^\circ = 12.856 \text{ m/s}$

② @ $t = 1.1 \text{ s}$: $v_x = x/t \Rightarrow x = v_x t = (15.321)(1.1) = \boxed{16.853 \text{ m}}$
 $y = v_{0y} t + \frac{1}{2} a t^2 = (12.856)(1.1) - 4.9(1.1)^2 = \boxed{8.213 \text{ m}}$

③ @ $t = 1.8 \text{ s}$: $x = (15.321)(1.8) = \boxed{27.578 \text{ m}}$
 $y = (12.856)(1.8) - 4.9(1.8)^2 = \boxed{7.265 \text{ m}}$

④ @ $t = 5 \text{ s}$: $x = (15.321)(5) = \boxed{76.605 \text{ m}}$
 $y = (12.856)(5) - 4.9(5)^2 = \boxed{-58.220 \text{ m}}$

∴ catapult should be shot from a cliff about 60 m high so that it is still traveling at $t = 5 \text{ s}$

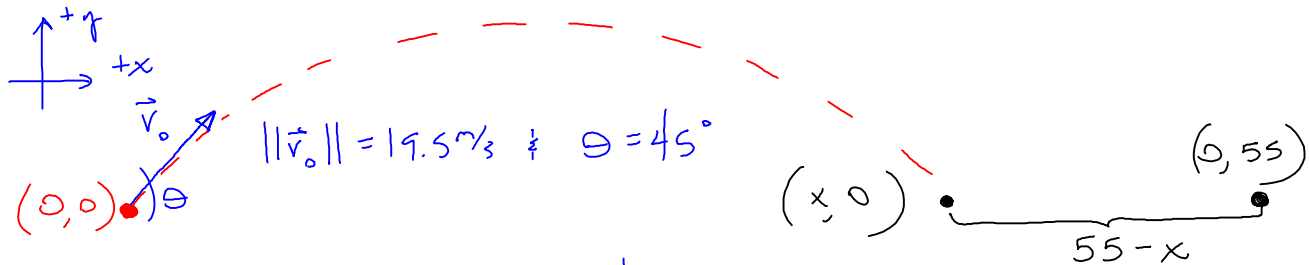
⑤ * If fired on level ground, what's the impact time?

$$y = v_{0y} t + \frac{1}{2} a t^2 \Rightarrow 0 = (12.856)t - 4.9t^2$$

$$= \underbrace{(12.856 - 4.9t)}_t t$$

$$\boxed{t = 2.624 \text{ s}}$$

••34 A soccer ball is kicked from the ground with an initial speed of 19.5 m/s at an upward angle of 45° . A player 55 m away in the direction of the kick starts running to meet the ball at that instant. What must be his average speed if he is to meet the ball just before it hits the ground?



$$\textcircled{1} \quad \begin{aligned} v_x &= v_0 \cos \theta = 19.5 \cos 45^\circ = 13.789 \text{ m/s} \\ v_{0y} &= v_0 \sin \theta = 19.5 \sin 45^\circ = 13.789 \text{ m/s} \end{aligned}$$

$$\textcircled{2} \quad y = v_{0y} t + \frac{1}{2} a t^2 \Rightarrow 0 = 13.789 t - 4.9 t^2$$

$$= (13.789 - 4.9 t) t$$

$$t = 2.814 \text{ s}$$

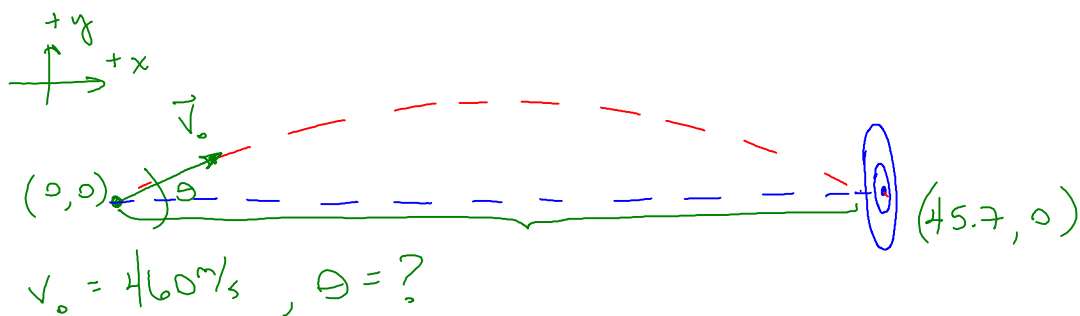
$$\textcircled{3} \quad v_x = \frac{x}{t} \Rightarrow x = v_x t = (13.789)(2.814) = 38.802 \text{ m}$$

$$55 \text{ m} - 38.802 \text{ m} = 16.198 \text{ m}$$

④ the player has 2.814 s to traverse 16.198 m

$$\bar{v} = \frac{16.198 \text{ m}}{2.814 \text{ s}} = \boxed{5.756 \text{ m/s}}$$

••39 A rifle that shoots bullets at 460 m/s is to be aimed at a target 45.7 m away. If the center of the target is level with the rifle, how high above the target must the rifle barrel be pointed so that the bullet hits dead center? **SSM**

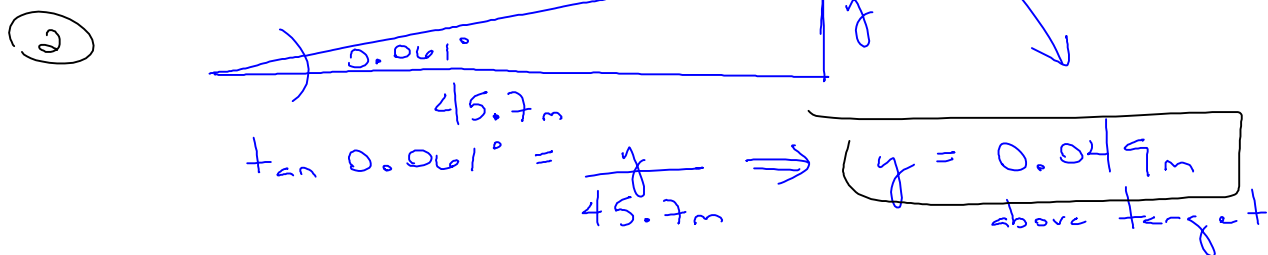


$$\textcircled{1} R = \frac{v_0^2 \sin 2\theta}{|g|} \Rightarrow \sin 2\theta = \frac{R|g|}{v_0^2}$$

$$\Rightarrow \theta = \frac{1}{2} \arcsin \left(\frac{R|g|}{v_0^2} \right)$$

$$= \frac{1}{2} \arcsin \left(\frac{45.7 \cdot 9.8}{460^2} \right)$$

$$= \boxed{0.061^\circ} \text{ above } +\hat{x}$$



OR derive range equation

$$v_x = \frac{x}{t} \Rightarrow t = \frac{x}{v_x} = \frac{x}{v_0 \cos \theta}$$

$$y = v_{0y} t + \frac{1}{2} a t^2 \Rightarrow 0 = v_0 \sin \theta \cdot \frac{x}{v_0 \cos \theta} + \frac{1}{2} a \left(\frac{x}{v_0 \cos \theta} \right)^2$$

$y=0$

$$\rightarrow 0 = x \frac{\sin \theta}{\cos \theta} + \frac{1}{2} a \frac{x^2}{v_0^2 \cos^2 \theta}$$

multiply by $\cos^2 \theta$
 v_0^2 and a
divide by x & a

$$0 = \frac{v_0^2}{a} \cdot \underbrace{2 \sin \theta \cos \theta}_{\sin 2\theta} + x$$

$$0 = \frac{v_0^2}{a} \sin 2\theta + x \Rightarrow \sin 2\theta = -\frac{x a}{v_0^2}$$

•••51 A football kicker can give the ball an initial speed of 25 m/s. What are the (a) least and (b) greatest elevation angles at which he can kick the ball to score a field goal from a point 50 m in front of goalposts whose horizontal bar is 3.44 m above the ground? **SSM**

$$\textcircled{1} \quad v_x = \frac{x}{t} \Rightarrow v_0 \cos \theta = \frac{x}{t}$$

$$25 \cos \theta = \frac{50}{t} \Rightarrow t = \frac{2}{\cos \theta}$$

$$\textcircled{2} \quad y = v_{0y} t + \frac{1}{2} a t^2 \Rightarrow 3.44 = v_0 \sin \theta t + \frac{1}{2} a t^2$$

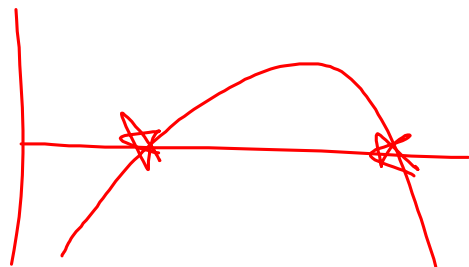
$$3.44 = 25 \sin \theta \cdot \frac{2}{\cos \theta} - 4.9 \left(\frac{2}{\cos \theta} \right)^2$$

$$3.44 = 50 \tan \theta - \frac{19.6}{\cos^2 \theta}$$

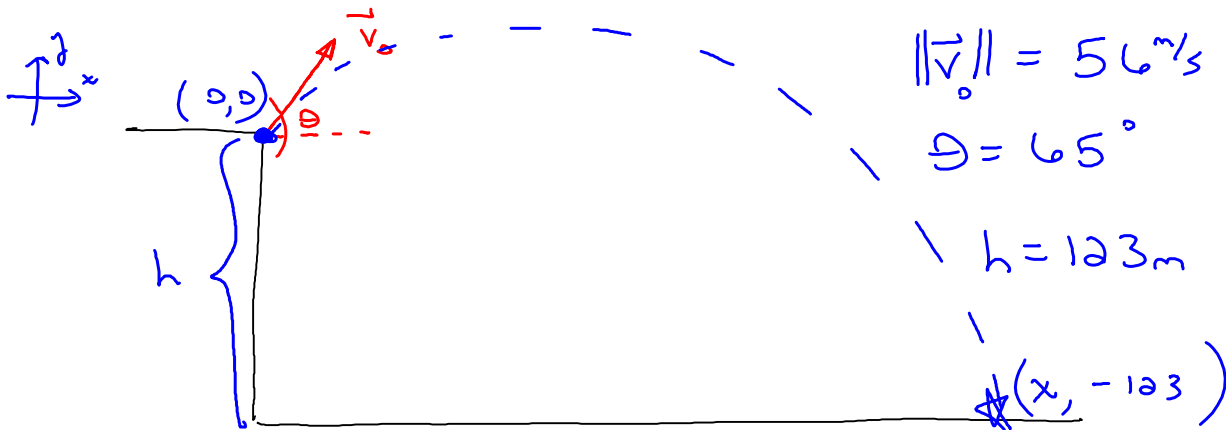
③ graph $Y_1 = 50 \tan X - \frac{19.6}{\cos^2 X} - 3.44$, find zeros using calculator

→ $\theta_{\min} = 31.117^\circ$ and $\theta_{\max} = 62.819^\circ$ ←

on the interval $[0, \pi/2]$



Projectile Motion Case III: fired at an angle from a cliff



① $v_x = v_0 \cos \theta = 56 \cos 65^\circ = 23.667 \text{ m/s}$

$v_{0y} = v_0 \sin \theta = 56 \sin 65^\circ = 50.753 \text{ m/s}$

② $y = v_{0y} t + \frac{1}{2} a t^2 \Rightarrow -123 = 50.753 t - 4.9 t^2$
 $4.9 t^2 - 50.753 t - 123 = 0$

$t = 12.385 \text{ s}$

③ $v_x = \frac{x}{t} \Rightarrow x = v_x t = (23.667)(12.385)$
 $= 293.116 \text{ m}$

④ $y = \frac{v^2 - v_{0y}^2}{2a} = \frac{-(50.753)^2}{2(-9.8)} = 131.422 \text{ m}$

max height above ground $123 \text{ m} + 131.422 \text{ m} = 254.422 \text{ m}$

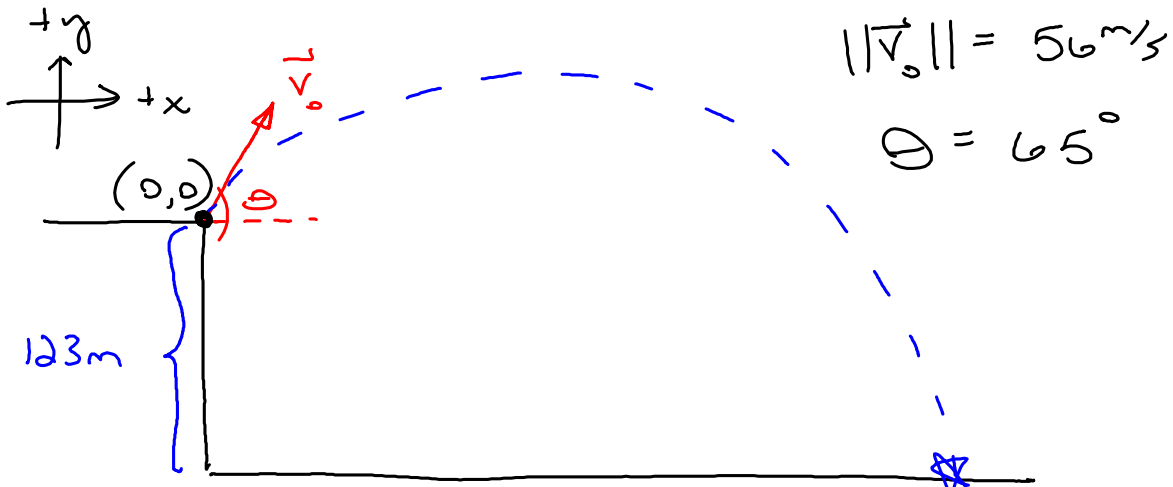
$v_x^2 + v_y^2 = v^2 \rightarrow 23.667^2 + v_y^2 = 56^2$

$$y = v_0 y t - \frac{1}{2} a t^2 \rightarrow 121.700 - 50.753 t - 7.1 t^2$$
$$4.9 t^2 - 50.753 t + 131 = 0$$

$$t = 4.885s \quad \text{;} \quad t = 5.472s$$

about 5.2s

CASE III: fired at an angle from a cliff



$$\textcircled{1} \quad v_x = v_0 \cos \Theta = 56 \cos 65^\circ = 23.667 \text{ m/s} \quad (x, -123)$$

$$v_{0y} = v_0 \sin \Theta = 56 \sin 65^\circ = 50.753 \text{ m/s}$$

$$\textcircled{2} \quad y = v_{0y} t + \frac{1}{2} a t^2 \Rightarrow -123 = 50.753 t - 4.9 t^2$$

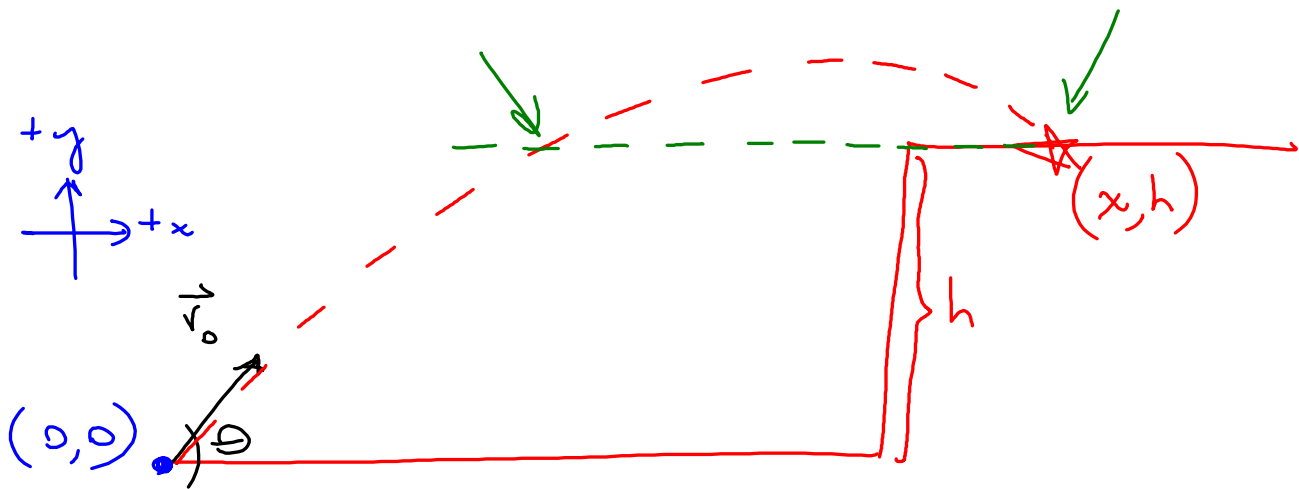
$$0 = -4.9 t^2 + 50.753 t + 123$$

use quadratic formula or graph: $t = 12.385 \text{ s}$

$$\textcircled{3} \quad v_x = \frac{x}{t} \Rightarrow x = v_x t = (23.667)(12.385)$$

$$= 293.116 \text{ m}$$

CASE IV: fired at an angle from the ground to the top of a cliff



HOMEWORK

HRW chapter 4 problems

#'s 26, 32, 38, 46, 109

old book \Rightarrow 22, 26, 30, 36, 80