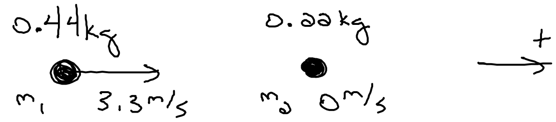
22. (II) A ball of mass 0.440 kg moving east ($+x$ direction) with a speed of $3.30\,\mathrm{m/s}$ collides head-on with a 0.220-kg ball at rest. If the collision is perfectly elastic, what will be the speed and direction of each ball after the collision?



$$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i} = \frac{0.44\,\mathrm{kg} - 0.22\,\mathrm{kg}}{0.44\,\mathrm{kg} + 0.22\,\mathrm{kg}} \cdot 3.3\,\mathrm{m/s} = \boxed{+1.1\,\mathrm{m/s}}$$

$$V_{2f} = \frac{2m_1}{m_1 + m_2} V_{1i} = \frac{2(0.44\,\mathrm{kg})}{0.44\,\mathrm{kg} + 0.22\,\mathrm{kg}} \cdot 3.3\,\mathrm{m/s} = \boxed{+4.4\,\mathrm{m/s}}$$

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23. (II) A 0.450-kg ice puck, moving east with a speed of 3.00 m/s, has a head-on collision with a 0.900-kg puck initially at rest. Assuming a perfectly elastic collision, what will be the speed and direction of each object after the collision?

$$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i} = \frac{0.45\,kg - 0.9\,kg}{0.45\,kg + 0.9\,kg} \cdot 3\,m/s = \boxed{-1\,m/s}$$

$$V_{2f} = \frac{2m_1}{m_1 + m_2} V_{1i} = \frac{2(0.45\,kg)}{0.45\,kg + 0.9\,kg} \cdot 3\,m/s = \boxed{+2\,m/s}$$

☆ end of page

24. (II) Two billiard balls of equal mass undergo a perfectly elastic head-on collision. If one ball's initial speed was $2.00\,\mathrm{m/s}$, and the other's was $3.00\,\mathrm{m/s}$ in the opposite direction, what will be their speeds after the collision?

$m_1 = m_2$

$$V_{1f} = \cancelto{0}{\frac{m_1 - m_2}{m_1 + m_2}} V_{1i} + \frac{2m_2}{m_1 + m_2} V_{2i}$$

$$= \frac{2m}{m+m} V_{2i} = \cancelto{1}{\frac{2m}{2m}} V_{2i} = \boxed{-3\,\mathrm{m/s}}$$

$$V_{2f} = \frac{2m_1}{m_1 + m_2} V_{1i} + \cancelto{0}{\frac{m_2 - m_1}{m_1 + m_2}} V_{2i}$$

$$= \frac{2m}{m+m} V_{1i} = \cancelto{1}{\frac{2m}{2m}} V_{1i} = \boxed{2\,\mathrm{m/s}}$$

the two objects have "exchanged velocities."
this will always be true for 1-D elastic collisions with equal masses!

★ end of page

31. (I) In a ballistic pendulum experiment, projectile 1 results in a maximum height h of the pendulum equal to 2.6 cm. A second projectile causes the the pendulum to swing twice as high, $h_2 = 5.2\,\text{cm}$. The second projectile was how many times faster than the first?

$h_1 = 0.026\,m$

$h_2 = 0.052\,m$

$$\boxed{v_i = \sqrt{2gh} \cdot \frac{m+M}{m}}$$

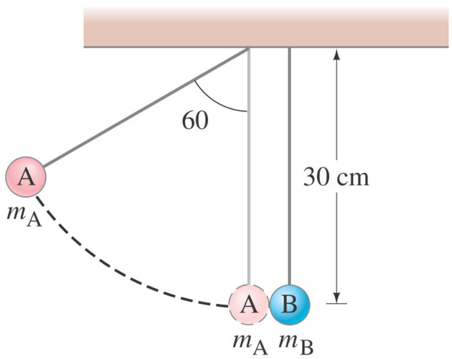
$$\frac{v_2}{v_1} = \frac{\sqrt{\cancel{2g}h_2} \cdot \cancel{\frac{m+M}{m}}}{\sqrt{\cancel{2g}h_1} \cdot \cancel{\frac{m+M}{m}}} = \frac{\sqrt{h_2}}{\sqrt{h_1}} = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{0.052\,m}{0.026\,m}} = \sqrt{2}$$

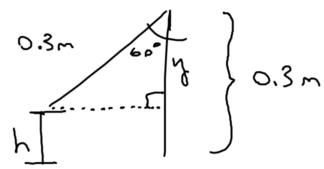
since $\frac{v_2}{v_1} = \sqrt{2} \Longrightarrow \boxed{v_2 = \sqrt{2}\, v_1}$

☆ end of page

76. Two balls, of masses $m_A = 40\,g$ and $m_B = 60\,g$, are suspended as shown in Fig. 7–44. The lighter ball is pulled away to a 60° angle with the vertical and released. (*a*) What is the velocity of the lighter ball before impact? (*b*) What is the velocity of each ball after the elastic collision? (*c*) What will be the maximum height of each ball after the elastic collision?

$40g = 0.04kg \qquad 60g = 0.06kg$





$h = 0.3m - y$

$\cos 60° = \frac{y}{0.3m} \Rightarrow y = 0.3m \cos 60° = 0.15m$

$h = 0.3m - 0.15m = 0.15m$

(a) $\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 \Rightarrow v = \sqrt{2gh}$
$= \sqrt{2(9.8\,m/s^2)(0.15m)}$
$= \boxed{1.715\,m/s}$

(b) $v_{Af} = \frac{m_A - m_B}{m_A + m_B} v_{Ai} = \frac{0.04kg - 0.06kg}{0.04kg + 0.06kg} \cdot 1.715\,m/s = \boxed{-0.343\,m/s}$

$v_{Bf} = \frac{2m_A}{m_A + m_B} v_{Ai} = \frac{2(0.04kg)}{0.04kg + 0.06kg} \cdot 1.715\,m/s = \boxed{1.372\,m/s}$

(c) $\cancel{m_A} g h_A = \frac{1}{2}\cancel{m_A} v_{Af}^2 \Rightarrow h_A = \frac{v_{Af}^2}{2g} = \frac{(-0.343\,m/s)^2}{2(9.8\,m/s^2)} = 0.0060025\,m$
$= \boxed{6.002 \times 10^{-3}\,m}$

$h_B = \frac{v_{Bf}^2}{2g} = \frac{(1.372\,m/s)^2}{2(9.8\,m/s^2)} = 0.09604\,m$
$= \boxed{9.604 \times 10^{-2}\,m}$

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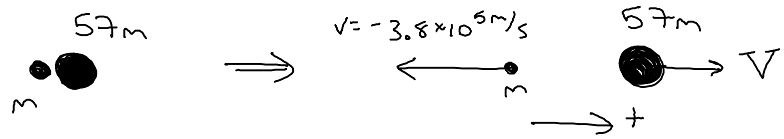
77. An atomic nucleus at rest decays radioactively into an alpha particle and a smaller nucleus. What will be the speed of this recoiling nucleus if the speed of the alpha particle is 3.8×10^5 m/s? Assume the recoiling nucleus has a mass 57 times greater than that of the alpha particle.



$$P_{before} = P_{after}$$

$$0 = m\vec{v} + M\vec{V}$$

$$0 = \cancel{m}(-3.8\times10^5 \text{ m/s}) + 57\cancel{m}V$$

$$V = +6666.667 \text{ m/s}$$
$$= \boxed{6.67\times10^3 \text{ m/s}}$$

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