11 OCT 11

• DAY 21 •

- MONDAY 10/17/11 ⇒ TEST # 2
  PROBLEM SET # 2 DUE
- 23.9 (due FRIDAY)

§23.9—Higher Derivatives

Let $y = 5x^3 - 2x$ and find $y' = \frac{dy}{dx}$ (this is called the ***first*** derivative).

$15x^2 - 2$

Now take the derivative of the derivative to find $y'' = \frac{d^2y}{dx^2}$ (this is called the ***second*** derivative).

$30x$

Differentiate again to find $y''' = \frac{d^3y}{dx^3}$

30

Differentiate again to find $y^{(4)} = \frac{d^4y}{dx^4}$

0

Remark: If $f(x)$ is a polynomial of degree *n*, then it will have *n* non-zero derivatives.

Example 2.

Find the higher derivatives of $f(x) = x(x^2-1)^2$

$f = x \Rightarrow f' = 1$

$g = (x^2-1)^2 \Rightarrow g' = 2(x^2-1)^1 \cdot 2x$

$= 4x(x^2-1)$

$gf' + fg' = (x^2-1)^2 \cdot 1 + x \cdot 4x(x^2-1)$

$= (x^2-1)^2 + 4x^2(x^2-1)$

$= (x^2-1)\left[(x^2-1) + 4x^2\right]$

$\boxed{f'(x) = (x^2-1)(5x^2-1)} = 5x^4 - 6x^2 + 1$

Example 3.

Find $f''(-2)$ for $f(x) = \dfrac{2}{1-x}$

$f(x) = 2(1-x)^{-1}$

$f'(x) = -2(1-x)^{-2} \cdot -1$

$= 2(1-x)^{-2}$

$f''(x) = -4(1-x)^{-3} \cdot -1$

$= 4(1-x)^{-3} = \dfrac{4}{(1-x)^3}$

$f''(-2) = \dfrac{4}{(1--2)^3} = \dfrac{4}{3^3} = \boxed{\dfrac{4}{27}}$

$f''(x) = 20x^3 - 12x$

$f'''(x) = 60x^2 - 12$

$f^{(4)}(x) = 120x$

$f^{(5)}(x) = 120$

$f^{(6)}(x) = 0$

Example 4.

Find y'' for $2x^2 + 3y^2 = 6$

$$4x + 6y \cdot \frac{dy}{dx} = 0$$

$$6y \frac{dy}{dx} = -4x$$

$$\frac{dy}{dx} = \frac{-4x}{6y} = \frac{-2x}{3y} \Rightarrow \boxed{\frac{dy}{dx} = \frac{-2x}{3y}}$$

$$4 + \frac{dy}{dx} \cdot 6\frac{dy}{dx} + 6y\frac{d^2y}{dx^2} = 0$$

$$4 + 6\left(\frac{dy}{dx}\right)^2 + 6y\frac{d^2y}{dx^2} = 0$$

$$4 + 6\left(\frac{-2x}{3y}\right)^2 + 6y\frac{d^2y}{dx^2} = 0$$

$$4 + \cancel{6}^{2}\left(\frac{4x^2}{\cancel{9}_{3}y^2}\right) + 6y\frac{d^2y}{dx^2} = 0$$

$$4 + \frac{8x^2}{3y^2} + 6y\frac{d^2y}{dx^2} = 0$$

$$3y^2 \cdot 6y\frac{d^2y}{dx^2} = \left(-4 - \frac{8x^2}{3y^2}\right)3y^2$$

$$18y^3\frac{d^2y}{dx^2} = -12y^2 - 8x^2 = -4\left(3y^2 + 2x^2\right)$$

$$\frac{d^2y}{dx^2} = \frac{-\cancel{4}^{2}\overbrace{\left(3y^2 + 2x^2\right)}^{6}}{\cancel{18}_{9}y^3}$$

$$= -\frac{2(6)}{4y^3} = \frac{-12}{4y^3} = \frac{-4}{3y^3}$$