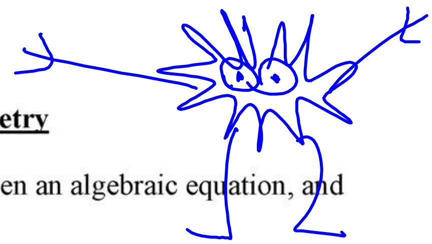
12 SEP 11

Day 1

- Section 21.1
- Expectations



§21.1—Basic Definitions in Plane Analytic Geometry

Analytic geometry deals with the relationship between an algebraic equation, and the geometric curve it represents.

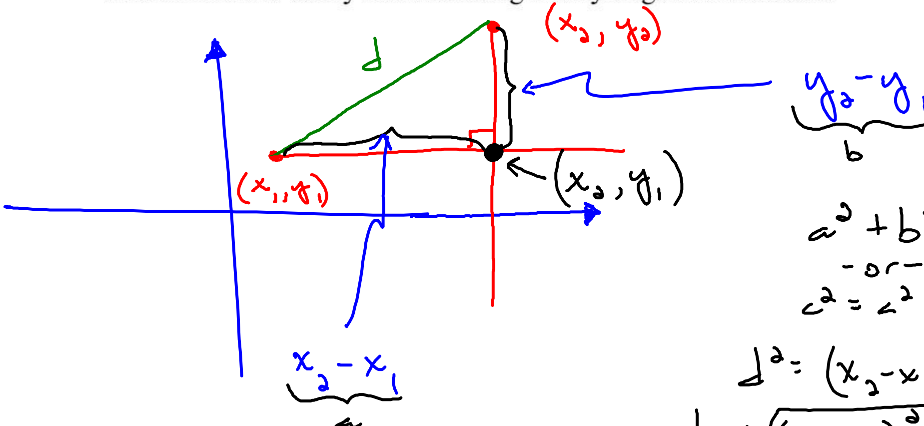
The Distance Formula

The distance between the points (x_1, y_1) and (x_2, y_2) is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

★MEMORIZE THIS!

This formula is easily derived using the Pythagorean theorem.



$a^2 + b^2 = c^2$

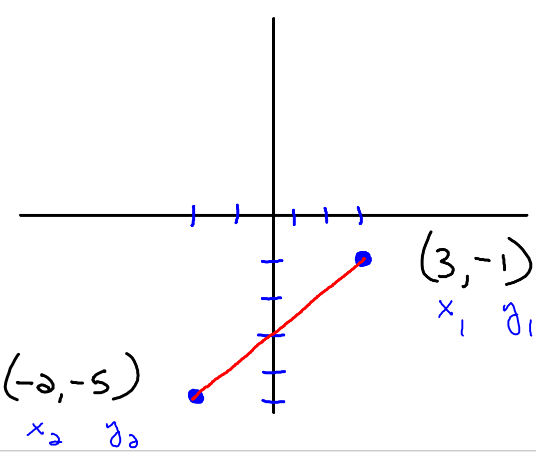
-or-

$c^2 = a^2 + b^2$

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example 1. Find the distance between $(3, -1)$ and $(-2, -5)$.



$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-2 - 3)^2 + (-5 - -1)^2}$

$= \sqrt{(-5)^2 + (-4)^2}$

$= \sqrt{25 + 16} = \sqrt{41}$

≈ 6.403

The Slope of a Line

The **slope** of a line through two points (x_1, y_1) and (x_2, y_2) is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{ryse}}{\text{run}}$$

Remark: a line rising to the right has a positive slope, and a line falling to the right has a negative slope.

Example 2. Find the slope of the line through $(3, -5)$ and $(-2, -6)$.

$x_1 \; y_1 \qquad x_2 \; y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - -5}{-2 - 3} = \frac{-1}{-5} = \frac{1}{5}$$

Inclination of a Line

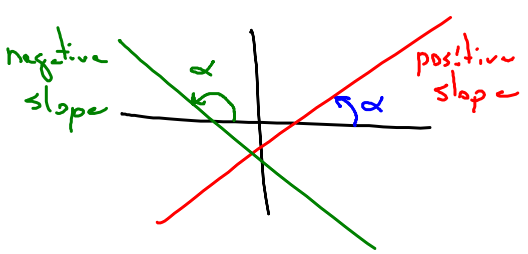
The **inclination** of a line is the angle measured from the *x*-axis in a positive direction to the line. The inclination of a line parallel to the *x*-axis is defined to be zero.

From this definition, we can easily derive the following alternate definition of slope, for a line with inclination α:

α = "alpha"

$$m = \tan\alpha \quad (0^\circ \le \alpha < 180^\circ)$$

calculator should be in degree mode



Example 3. Find the slope of a line with an inclination of 45°.

$m = \tan\alpha \Rightarrow m = \tan 45^\circ$ ← degree mode

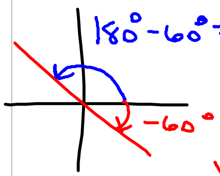
$m = 1$

Example 4. Find the inclination of a line that has a slope of -1.732.

$m = \tan\alpha \Rightarrow \alpha = \tan^{-1}\alpha$

$-1.732 = \tan\alpha$

again, DEGREE mode



$\alpha^* = \tan^{-1}(-1.732)$

$\alpha^* = -60^\circ$

WE WANT A POSITIVE ANGLE!

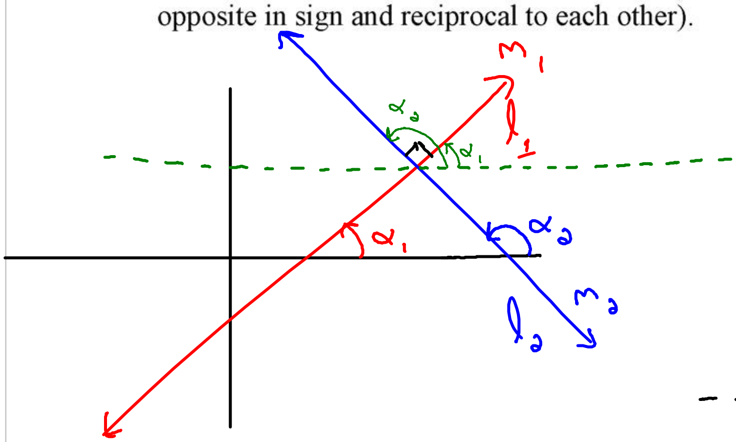
$\alpha = 120^\circ$

Remarks: If α is acute, then the slope is positive, and if α is obtuse, then the slope is negative.

Parallel and Perpendicular Lines

Two lines are **parallel** if they have the same slope ($m_1 = m_2$).

Two lines are **perpendicular** if they meet at 90°. From this fact we can derive that if two lines are perpendicular, then $m_2 = -\frac{1}{m_1}$ or $m_1 m_2 = -1$ (their slopes are opposite in sign and reciprocal to each other).



RECALL: tangent is a odd function

tangent & cotangent are cofunctions

$$l_1 \perp l_2 \Rightarrow \alpha_2 - \alpha_1 = 90^\circ$$

$$-\alpha_2 \qquad\qquad -\alpha_2$$

$$-\alpha_1 = 90^\circ - \alpha_2$$

$$\tan(-\alpha_1) = \tan(90^\circ - \alpha_2)$$

$$-\tan(\alpha_1) = \cot(\alpha_2)$$

$$-\tan(\alpha_1) = \frac{1}{\tan(\alpha_2)}$$

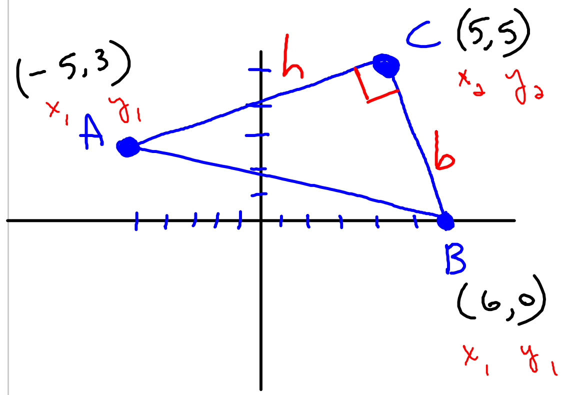
$$-m_1 = \frac{1}{m_2}$$

-OR-

$$m_1 = -\frac{1}{m_2}$$

$$m_1 m_2 = -1$$

Example 5. Show that the line segments joining $A(-5,3)$, $B(6,0)$ and $C(5,5)$ form a right triangle. Then find its area.



WANT

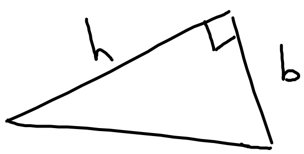
$$m_{AC} = \frac{-1}{m_{BC}}$$

$$m_{AC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{5--5} = \frac{2}{10} = \frac{1}{5}$$

$$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-0}{5-6} = \frac{5}{-1}$$

$$\frac{1}{5} \cdot \frac{-5}{1} = -1 \ \checkmark$$

$$A = \frac{1}{2}bh$$



$$b = d_{BC} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(5-6)^2 + (5-0)^2}$$
$$= \sqrt{(-1)^2 + (5)^2} = \sqrt{1+25} = \sqrt{26}$$

$$h = d_{AC} = \sqrt{(5+5)^2 + (5-3)^2}$$
$$= \sqrt{(10)^2 + (2)^2} = \sqrt{104}$$
$$= \sqrt{4 \cdot 26}$$
$$= 2\sqrt{26}$$

$$A = \frac{1}{2}bh = \frac{1}{2}(\sqrt{26})(2\sqrt{26}) = \boxed{26}$$

Monday, September 12, 2011
1:33 PM

Homework

⇒ 21.1 #'s 5 → 47

every other odd (E.O.O.)

i.e. 5, 9, 13, 17... etc

also do 55 & 57

⇒ signed syllabus

⇒ email Rachel to say "hi"

⇒ complete registration form