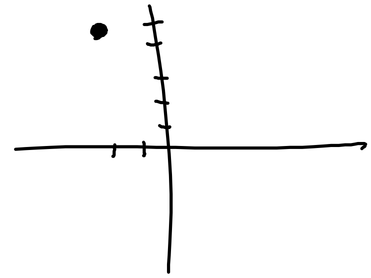
21.1 57 $m = 3$ $(-2, 5)$

WANT: other endpoint on x-axis

$\therefore$ y value must be zero

$(x, 0)$



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-5}{x++2} = 3 \Rightarrow \frac{-5}{x+2} = 3$$

$$-5 = 3(x+2)$$

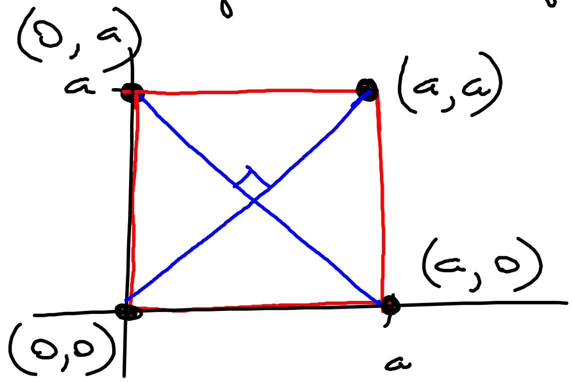
$$-5 = 3x + 6$$

$$3x = -11$$

$$x = -\frac{11}{3}$$

so... $\left(-\frac{11}{3}, 0\right)$

(55) Show that diagonals of a square are perpendicular



$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{a - 0}{a - 0} = 1$$

$$m_2 = \frac{a - 0}{0 - a} = \frac{a}{-a} = -1$$

$$m_1 = \frac{-1}{m_2}$$

§21.2—The Straight Line

The Slope-Intercept Form of a line is $y = mx + b$ where m is the slope of the line, and b is the y-intercept.

This form of the equation of a line is useful for graphing lines.

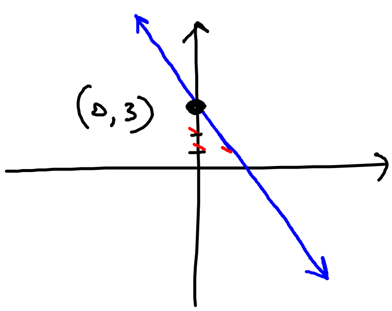
Example 1. Sketch the graph of the line $2y + 4x - 6 = 0$.

$$\frac{2y}{2} = \frac{-4x}{2} + \frac{6}{2}$$

$$y = -2x + 3 \Rightarrow m = -2$$

$$b = 3$$

$(0, 3)$



The Point-Slope Form of a line is $y - y_1 = m(x - x_1)$ where m is the slope, and (x_1, y_1) is a known point on the line.

This form is useful for finding the equation of a line.

$x_1 \quad y_1 \qquad x_2 \quad y_2$

Example 2. Find the equation of the line passing through $(2,-1)$ and $(6,2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 + +1}{6 - 2} = \frac{3}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y + +1 = \frac{3}{4}(x - 2) \Rightarrow y + 1 = \frac{3}{4}x - \frac{3}{2}$$

$$y = \frac{3}{4}x - \frac{3}{2} - 1 \qquad 1 = \frac{2}{2}$$

$$\boxed{y = \frac{3}{4}x - \frac{5}{2}}$$

Observations: A *vertical* line has equation $x = a$
A *horizontal* line has equation $y = b$

The General Form of a line is

$$Ax + By + C = 0$$

where A, B, and C are $\boxed{\text{integers.}}$ $\Rightarrow$ no fractions!

Example 3. Find the general form of the equation of the line passing through $(-1,2)$ and parallel to the line $3x + 2y - 6 = 0$.

find the slope $\Rightarrow$ $3x + 2y - 6 = 0$

$$\frac{2y}{2} = \frac{-3x}{2} + \frac{6}{2}$$

$$y = -\frac{3}{2}x + 3 \qquad \therefore m = -\frac{3}{2}$$

$$y - y_1 = m(x - x_1)$$

$$2(y - 2) = \left(-\frac{3}{2}(x + 1)\right)2$$

$$2y - 4 = -3(x + 1)$$

$$2y - 4 = -3x - 3$$

$$\boxed{3x + 2y - 1 = 0}$$

OR! $-3x - 2y + 1 = 0$

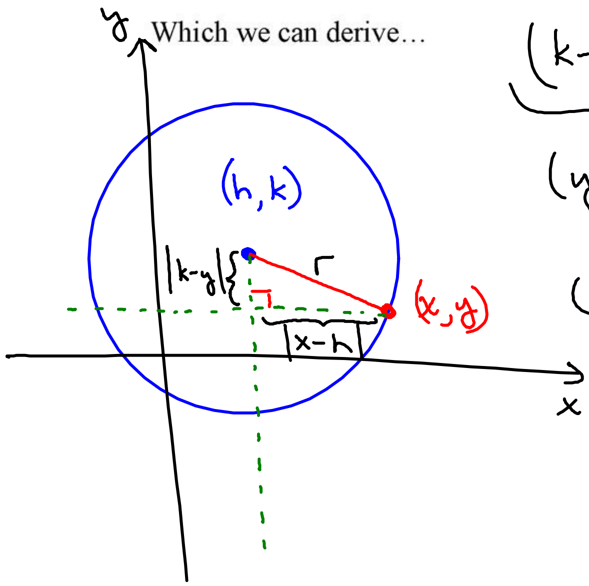
§21.3 – The Circle

Definition: A **circle** is the set of points that are a fixed distance, r (called the radius) from a fixed center point (h,k).

The **standard form** of a circle with center at (h,k) and radius r is:

$$(x-h)^2+(y-k)^2=r^2$$

Which we can derive…



$$\underbrace{(k-y)^2+(h-x)^2=r^2}$$

$$(y-k)^2+(x-h)^2=r^2$$

$$(x-h)^2+(y-k)^2=r^2$$

Example 1.

Find the center and radius of the circle $(x-1)^2+(y+2)^2=16 \leftarrow r^2$

$h=1 \qquad k=-2$

if $r^2=16 \Rightarrow r=4$

center: $(h,k) \Rightarrow (1,-2)$ with $r=4$

Example 2.

h k

Find the equation of a circle with center (2,1) and which passes through the point (4,8).

x y

$$(x-h)^2+(y-k)^2=r^2$$

$$(x-2)^2+(y-1)^2=r^2$$

$$(4-2)^2+(8-1)^2=r^2$$

$$(2)^2+(7)^2=r^2$$

$$4+49=r^2 \Rightarrow r^2=53$$

So, $\boxed{(x-2)^2+(y-1)^2=53}$

Completing the Square coefficients need to be 1 !

The **general form** of an equation of a circle is $x^2 + y^2 + Dx + Ey + F = 0$. To write this equation in standard form, we must complete the square on both *x* and *y*.

Here's the general procedure:

1. Get the constant alone on the other side, and group the *x*'s and *y*'s together.
2. If necessary, divide both sides by the coefficient of x^2 and y^2.
3. Add the appropriate constants (on *both* sides) so that we have a perfect square for both *x* and *y* terms.
4. Re-write the left hand side so that the equation is in standard form.

Example 3.

Complete the square on the equation below to write it in standard form.

$x^2 + y^2 - 6x + 8y - 24 = 0$

step 1 $x^2 - 6x \quad + y^2 + 8y \quad = 24$

step 2 not needed

step 3 $\underbrace{x^2 - 6x + (-3)^2}_{} + \underbrace{y^2 + 8y + (4)^2}_{} = \underbrace{24 + (-3)^2 + (4)^2}_{}$

$\frac{-6}{2} = -3 \qquad \frac{8}{2} = 4$

step 4 $\boxed{(x-3)^2 + (y+4)^2 = 49}$

$\left.\begin{array}{l} h = 3 \\ k = -4 \end{array}\right\}$ center @ $(3, -4)$

$r^2 = 49 \Longrightarrow r = 7$

Example 4.

Write the equation of the circle below in standard form.

$$\frac{4x^2}{4} + \frac{4y^2}{4} - \frac{16y}{4} = \frac{9}{4} \Longrightarrow x^2 + y^2 - 4y = \frac{9}{4}$$

$$x^2 + y^2 - 4y + (-2)^2 = \frac{9}{4} + (-2)^2$$

$$^*\ \underbrace{\frac{9}{4} + 4}$$

$$\underbrace{\frac{9}{4} + \frac{4}{1}\cdot\frac{4}{4}}$$

$$\frac{9}{4} + \frac{16}{4} = \frac{25}{4}$$

$$(x-0)^2 + (y-2)^2 = \frac{25}{4}\ ^*$$

OR $x^2 + (y-2)^2 = \dfrac{25}{4}$

center @ $(0, 2)$ with $r = \dfrac{5}{2}$

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Homework

21.2 DUE THURSDAY

5-39 E.O.O., 43

21.3 DUE THURSDAY

5-35 E.O.O., 41, 54

21.7 #39 DUE THURSDAY

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MAT 1520 TV1

Calculus for Engineering

VIT

24

$1,708

$50

$$(x_2 - x_1)\,m = \underline{(y_2 - y_1)}\,\cancel{(x_2 - x_1)}$$

$$\cancel{x_2 - x_1}$$

$$y_2 - y_1 = m(x_2 - x_1)$$

$$\sqrt{8} = 2\sqrt{2}$$

$y = mx + b$

$y - y_1 = m(x - x_1)$

$Ax + By + C = 0$

$y - 3 = 4(x + 2)$

$b = 4 \Rightarrow (0,4)$