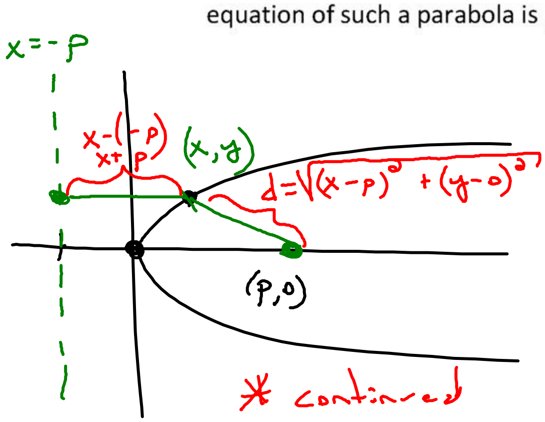
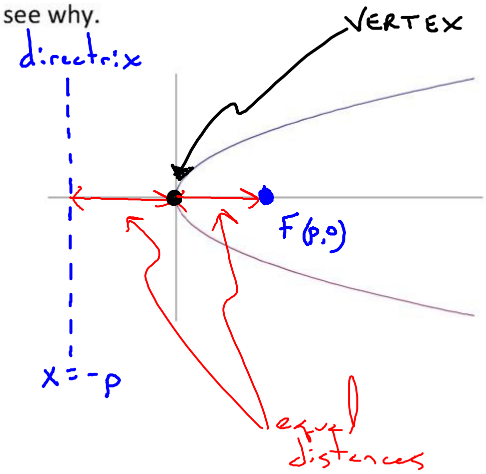
§21.4—The Parabola

Definition: A **parabola** is the set of points equidistant from a given line (called the **directrix**) and a given fixed point (called the **focus**). The point midway between the focus and directrix is called the **vertex**, and the line perpendicular to the directrix that goes through the vertex is called the **axis** (or axis of symmetry).

Parabolas With Their Vertex at the Origin

I. If the focus of a parabola is $(p,0)$ and the directrix is the line $x=-p$, then the vertex is at the origin, and the axis is the x-axis. The **standard form** for the equation of such a parabola is $y^2=4px$. Let's see why.





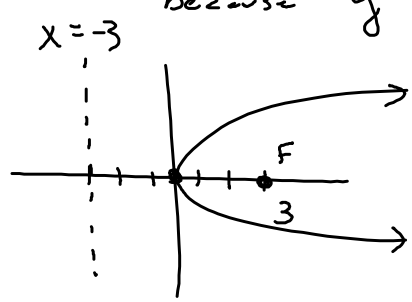
Example 1. Find the focus and directrix of the parabola $y^2=12x$. Then sketch it.

$y^2=12x$

$y^2=4(3)x$ extract the 4 because $y^2=4px$

$\therefore p=3$

$+ \Rightarrow$ opens right $\subset$

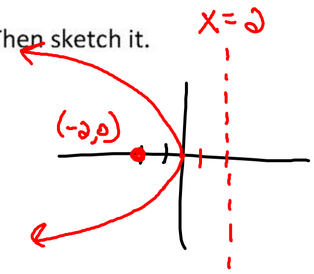


Example 2. Find the focus and directrix of the parabola $y^2 = -8x$. Then sketch it.

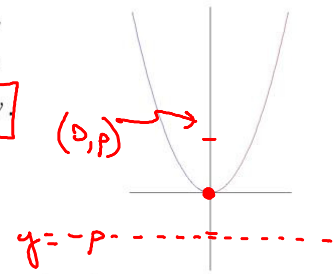
$y^2 = -8x \qquad y^2 = 4px$

$y^2 = 4(-2)x \Rightarrow p = -2$

$F(-2, 0) \Rightarrow$ directrix $x = 2$



II. If the focus of a parabola is $(0, p)$ and the directrix is $y = -p$, then the vertex is at the origin and the axis is the y-axis. The standard form for the equation of such a parabola is $\boxed{x^2 = 4py}$.



Example 3. Find the focus and directrix of the parabola $x^2 = 4y$. Then sketch it.

$x^2 = 4y$

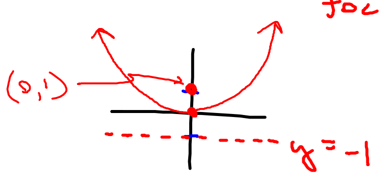
$x^2 = 4(1)y$

$p = 1$ which is positive

$\therefore$ it opens upwards

focus: $(0, 1)$ directrix $y = -1$

*p is always the distance from the focus to the vertex



if $y = 4 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

if $y = 1 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

Example 4. Find the focus and directrix of the parabola $2x^2 = -9y$. Then sketch it.

$\frac{2x^2}{2} = \frac{-9y}{2}$

$x^2 = -\frac{9}{2}y$

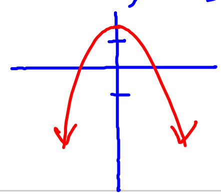
$x^2 = 4\left(-\frac{9}{8}\right)y$

$\therefore p = -\frac{9}{8}$

$4 \cdot ? = -\frac{9}{2}$

$? = -\frac{9}{2} \cdot \frac{1}{4} = -\frac{9}{8}$

focus: $(0, -9/8)$ directrix: $y = 9/8$



if $y = -2$

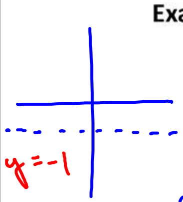
$2x^2 = -9(-2)$

$2x^2 = 18 \quad x^2 = 9$

Observation. We can tell we're looking at the equation of a parabola if one variable is squared and the other isn't. The parabola always opens about the non-squared variable.

Example 5. (*vertex not at the origin*) Use the definition of the parabola to find the equation of the parabola with focus $(2,3)$ and directrix $y=-1$.

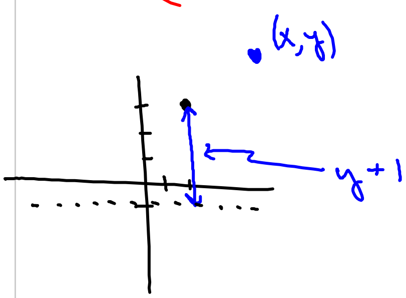
horizontal line



(x,y) is on the parabola

distance (2,3) = distance to y = -1

$$\underbrace{\left(\sqrt{(x-2)^2+(y-3)^2}\right)^2=(y+1)^2}$$



$$x^2-4x+4+\cancel{y^2}-6y+9=\cancel{y^2}+2y+1$$

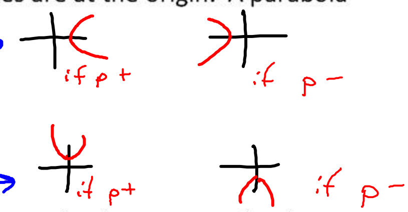
$$x^2-4x-8y+12=0$$

Translation of Axes

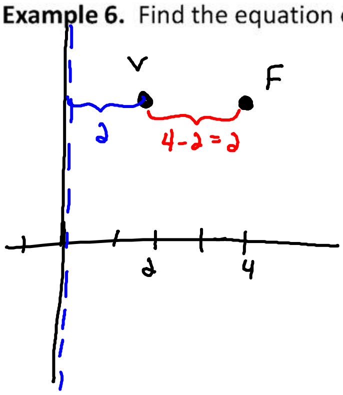
So far, we have only looked at parabolas whose vertices are at the origin. A parabola with vertex (h,k) has an equation of the form

$(y-k)^2 = 4p(x-h)$ (axis parallel to the x-axis)

$(x-h)^2 = 4p(y-k)$ (axis parallel to the y-axis)



Example 6. Find the equation of the parabola with vertex $(2,4)$ and focus $(4,4)$.



p is distance from V to F

$$(y-k)^2 = 4p(x-h)$$
$$(y-4)^2 = 4(2)(x-2)$$

Example 7. Cylindrical glass beakers are to be made with a height of 3 inches. Express the surface area in terms of the radius of the base and sketch the curve.

Wednesday, September 14, 2011
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* $$(x+p)^2 = \left(\sqrt{(x-p)^2 + (y-0)^2}\right)^2$$

$$x^2 + 2xp + p^2 = (x-p)^2 + y^2$$

$$\cancel{x^2} + 2xp + \cancel{p^2} = \cancel{x^2} - 2xp + \cancel{p^2} + y^2$$

$$4px = y^2 \Rightarrow \boxed{y^2 = 4px}$$

QED

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example

$$2x^2 - 4x - 9y = -2$$

$$\frac{2x^2}{2} - \frac{4x}{2} = \frac{9y}{2} - \frac{2}{2}$$

$$x^2 - 2x = \frac{9}{2}y - 1$$

$$x^2 - 2x + (-1)^2 = \frac{9}{2}y - 1 + (-1)^2$$

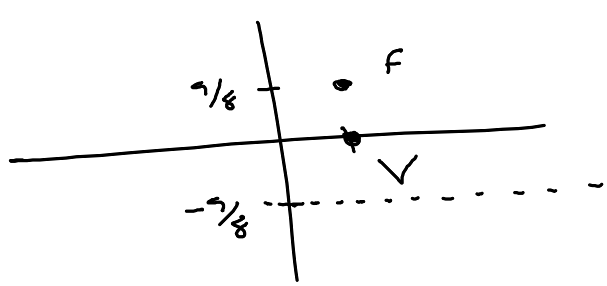
$$(x-1)^2 = \frac{9}{2}y$$

$$(x-h)^2 = 4p(y-k)$$ where k in this case is zero

$$(h,k) \Rightarrow (1,0)$$

$$4p = \frac{9}{2} \Rightarrow p = \frac{9}{8}$$

opens up



Homework

21.4 $\quad 5 \to 29$ odd

35, 37, 43, 45

DUE MONDAY

try to log into MOODLE

also for Monday

21.7

39 $9x^2 + 9y^2 + 14 = 6x + 24y$

$$\frac{9x^2}{9} - \frac{6x}{9} + \frac{9y^2}{9} - \frac{24y}{9} = \frac{-14}{9}$$

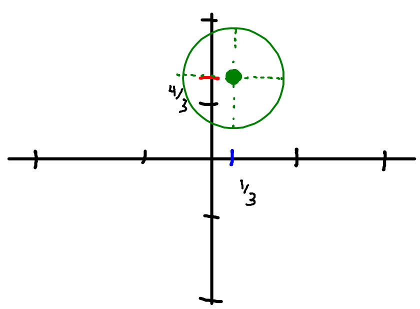
$$x^2 - \frac{2}{3}x + \left(-\frac{1}{3}\right)^2 + y^2 - \frac{8}{3}y + \left(-\frac{4}{3}\right)^2 = \frac{-14}{9} + \left(\frac{-1}{3}\right)^2 + \left(\frac{-4}{3}\right)^2$$

$\frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$ $\qquad \frac{8}{3} \cdot \frac{1}{2} = \frac{4}{3}$ $\qquad \frac{-14}{9} + \frac{1}{9} + \frac{16}{9} = \frac{3}{9} = \frac{1}{3}$

$$\left(x - \frac{1}{3}\right)^2 + \left(y - \frac{4}{3}\right)^2 = \frac{1}{3}$$

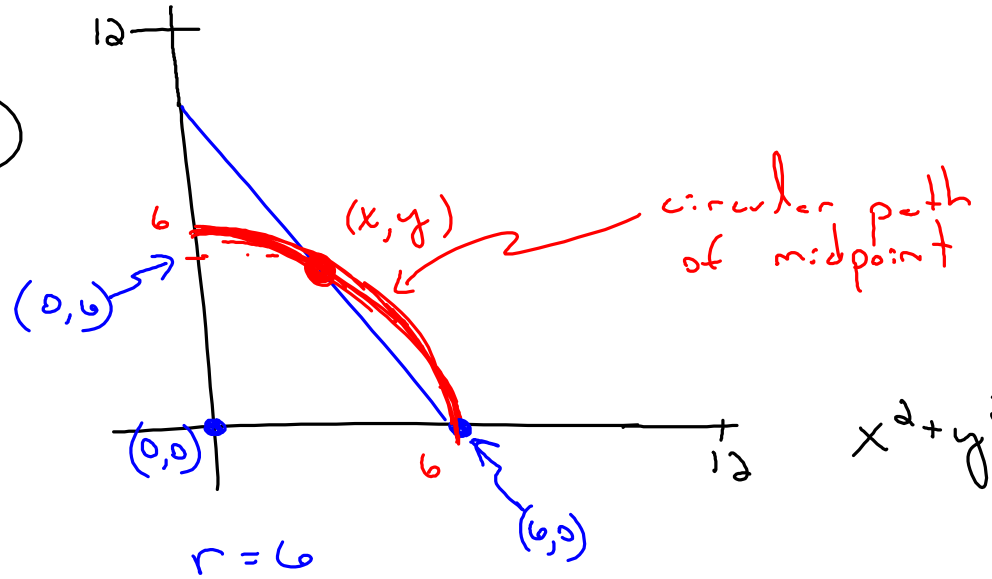
center @ $\left(1/3, 4/3\right)$

$$r^2 = \frac{1}{3} \Longrightarrow r = \sqrt{\frac{1}{3}} = \frac{\sqrt{1}}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} \approx 0.58$$



~~21.3~~

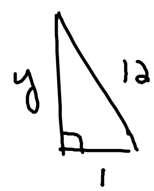
#54



$$x^2+y^2=36$$

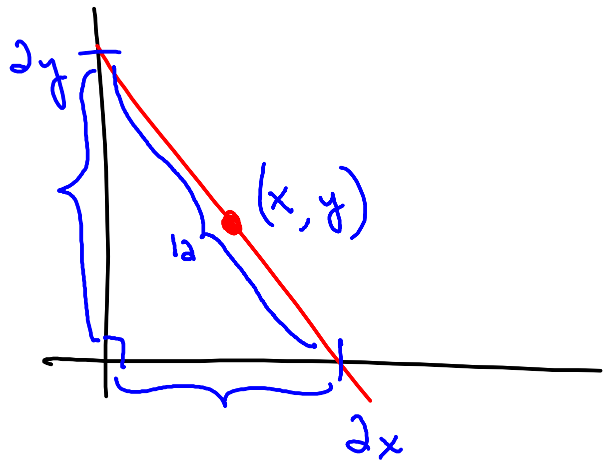
$r=6$

$\Rightarrow$ What if $x=1$?



$$y=\sqrt{12^2-1^2}=\sqrt{143}\approx 11.96$$

two points: $(1,0)$; $(0, 11.96)$



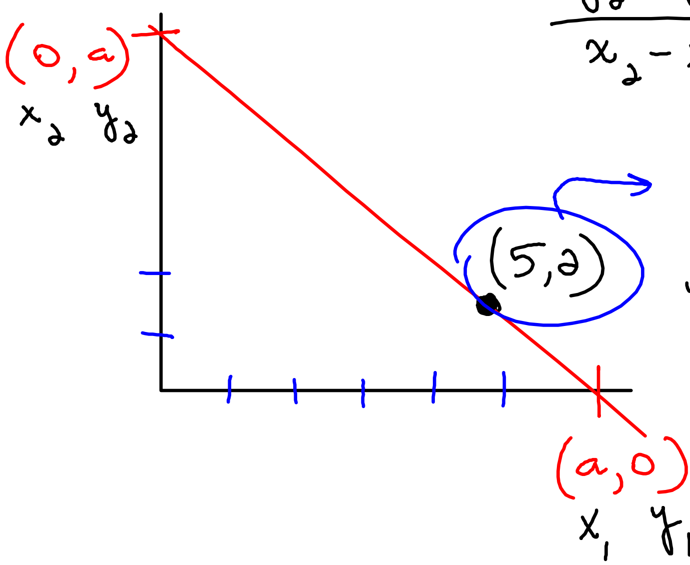
$$(2x)^2 + (2y)^2 = 12^2$$

$$\frac{4x^2}{4} + \frac{4y^2}{4} = \frac{144}{4}$$

$$x^2 + y^2 = 36$$

21.2

(17) eqn of line
equal intercepts
passes through $(5,2)$



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{a - 0}{0 - a} = -1$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -1(x - 5)$$

$$y - 2 = -x + 5$$

$$\boxed{y = -x + 7}$$