19 JAN 12

• DAY 77 •

Q10

- HW ✓ & Q&A with WARM-UP
- ΔK & bullets
- Work and the Elevator Problem
- Work done by a variable force
- Hooke's Law (?) OR CLICKERS ✓
- Homework Assignment

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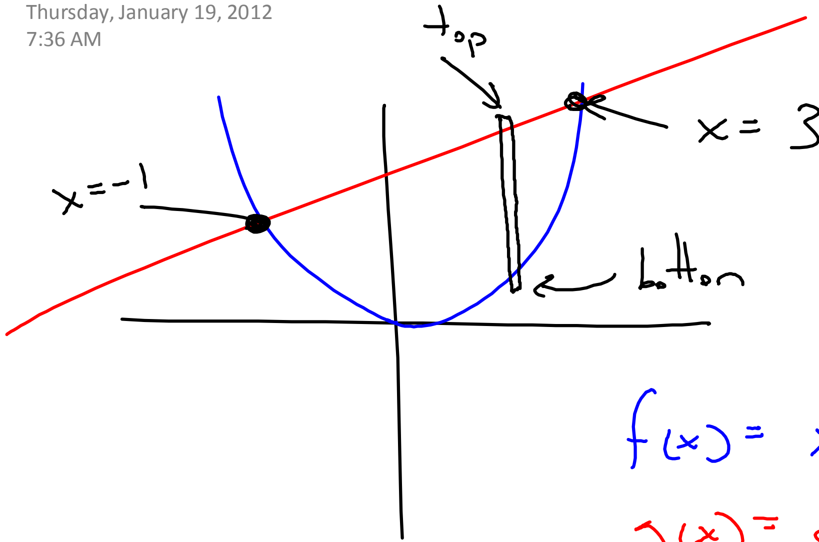
Thursday, January 19, 2012
7:36 AM



$f(x) = x^2$ bottom

$g(x) = 2x + 3$ top

$$x^2 = 2x + 3$$

$$x^2 - 2x - 3 = 0 \Rightarrow (x+1)(x-3) = 0$$

$$\int_{-1}^{3} \left[(2x+3) - (x^2) \right] dx$$

$$= \int_{-1}^{3} \left(2x + 3 - x^2 \right) dx$$

$$\left(x^2 + 3x - \frac{x^3}{3} \right) \Big|_{-1}^{3}$$

$$= \left[(3)^2 + 3(3) - \frac{(3)^3}{3} \right] - \left[(-1)^2 + 3(-1) - \frac{(-1)^3}{3} \right]$$

$$= \left[9 + \cancel{9} - \cancel{9} \right] - \left[1 - 3 + \frac{1}{3} \right]$$

$$= [9] - [-2 + \frac{1}{3}]$$

$$= 9 + 2 - \frac{1}{3} = \frac{3}{3}11 - \frac{1}{3}$$

$$= \frac{33}{3} - \frac{1}{3} = \boxed{\frac{32}{3}}$$

Warm-up

33. If $\frac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $\boxed{x = 2,\ y =}$

(A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) $\frac{2}{3}$

$$\frac{dy}{dx} = 2y^2 \Rightarrow dy = 2y^2\,dx$$

$$\frac{dy}{2y^2} = dx$$

$$\int \frac{1}{2} y^{-2}\,dy = \int dx$$

$$\frac{1}{2}\frac{y^{-1}}{-1} = x + C$$

$$-\frac{1}{2y} = x + C$$

$$-\frac{1}{2(-1)} = 1 + C$$

$$\frac{1}{2} = 1 + C$$

$$C = -\frac{1}{2}$$

$$-\frac{1}{2y} = x - \frac{1}{2}$$

$$-\frac{1}{2y} = 2 - \frac{1}{2}$$

$$\left(-\frac{1}{2y}\right)^{-1} = \left(\frac{3}{2}\right)^{-1}$$

$$-2y = \frac{2}{3}$$

$$y = -\frac{1}{3}$$

$$F(x) = \int_{-\pi}^{\cos x} 2t^3\,dt$$

$$F'(x) = \frac{d}{dx}\int_{-\pi}^{\cos x} 2t^3\,dt$$

$$2(\cos x)^3 \cdot -\sin x$$

$$\boxed{-2\cos^3 x \sin x}$$

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Thursday, January 19, 2012
8:09 AM

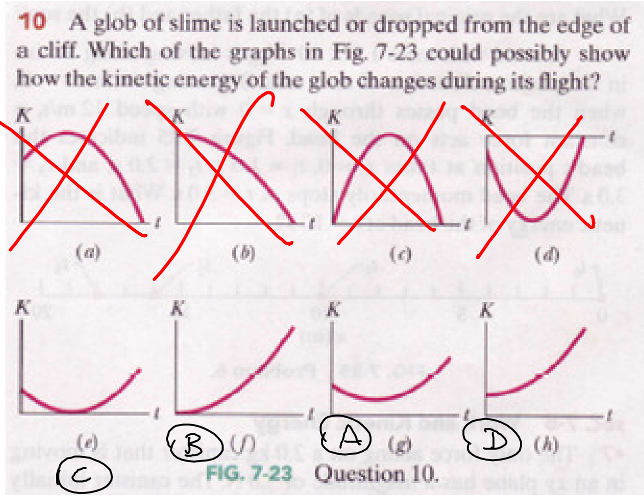
$$\int x\sqrt{x-1}\,dx$$

let $u = x-1 \rightarrow x = u+1$

$du = dx$

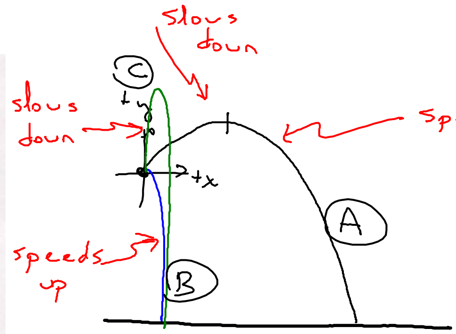
$$\int (u+1)\,u^{1/2}\,du = \int \left(u^{3/2} + u^{1/2}\right)du$$

q10

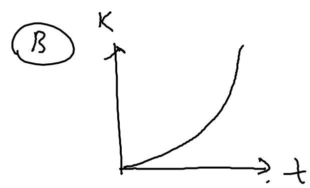


10 A glob of slime is launched or dropped from the edge of a cliff. Which of the graphs in Fig. 7-23 could possibly show how the kinetic energy of the glob changes during its flight?

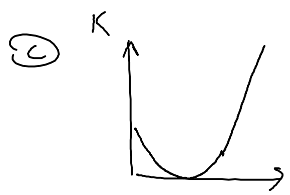
FIG. 7-23 Question 10.



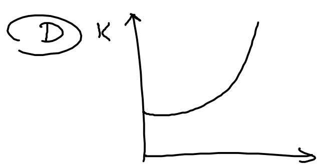
thrown at an angle



dropped



thrown straight up, the falls



thrown straight down

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•9 A 3.0 kg body is at rest on a frictionless horizontal air track when a constant horizontal force $\vec{F}$ acting in the positive direction of an x axis along the track is applied to the body. A stroboscopic graph of the position of the body as it slides to the right is shown in Fig. 7-26. The force $\vec{F}$ is applied to the body at $t = 0$, and the graph records the position of the body at 0.50 s intervals. How much work is done on the body by the applied force $\vec{F}$ between $t = 0$ and $t = 2.0$ s?

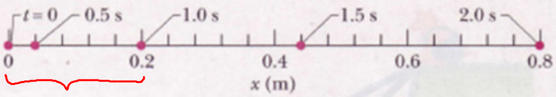

FIG. 7-26 Problem 9.

$a = ?$

$\Delta t = 1s$

$x = 0.2m$

$v_0 = 0$

$x = \cancel{v_0 t}^{\,0} + \frac{1}{2} a t^2$

$a = \frac{2x}{t^2} = \frac{2(0.2m)}{(1s)^2} = 0.4\, m/s^2$

$v = ?$ $\quad a = \frac{v - \cancel{v_0}^{\,0}}{t} \Rightarrow v = at$

$= (0.4\, m/s^2)(2s)$

$v = 0.8\, m/s$

$W = \Delta K = K_f - \cancel{K_i}^{\,0} = \frac{1}{2} m v^2 = \frac{1}{2}(3kg)(0.8\, m/s)^2 = \boxed{0.96\, J}$

or

$W = F_x d_x = m a_x d_x = (3kg)(0.4\, m/s^2)(0.8m) = \boxed{0.96\, J}$

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••11 A luge and its rider, with a total mass of 85 kg, emerge from a downhill track onto a horizontal straight track with an initial speed of 37 m/s. If a force slows them to a stop at a constant rate of 2.0 m/s², (a) what magnitude F is required for the force, (b) what distance d do they travel while slowing, and (c) what work W is done on them by the force? What are (d) F, (e) d, and (f) W if they, instead, slow at 4.0 m/s²?

$m = 85\,kg$

$v_0 = 37\,m/s$

$v = 0$

$a = -2\,m/s^2$

(a) $F = ma = (85\,kg)(-2\,m/s^2)$

$\|F\| = \boxed{170\,N}$

(b) $x = \dfrac{\cancel{v^2}^{\,0} - v_0^2}{2a} = \dfrac{-(37\,m/s)^2}{2(-2\,m/s^2)} = 342.25\,m$

thus, distance is $\boxed{342.25\,m}$

(c) $W = \vec{F} \cdot \vec{d} = (-170\,N)(342.25\,m)$

$= \boxed{-58{,}182.5\,J}$

if $a = -4\,m/s$

(d) $\|\vec{F}\| = 340\,N$

(e) $x = 171.125\,m$

(f) $W = 58182.5\,J$

* by doubling a,
we double F
but half x
∴ same result

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••21 In Fig. 7-32, a constant force $\vec{F}_a$ of magnitude 82.0 N is applied to a 3.00 kg shoe box at angle $\phi = 53.0°$, causing the box to move up a frictionless ramp at constant speed. How much work is done on the box by $\vec{F}_a$ when the box has moved through vertical distance $h = 0.150$ m?

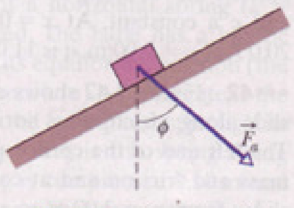
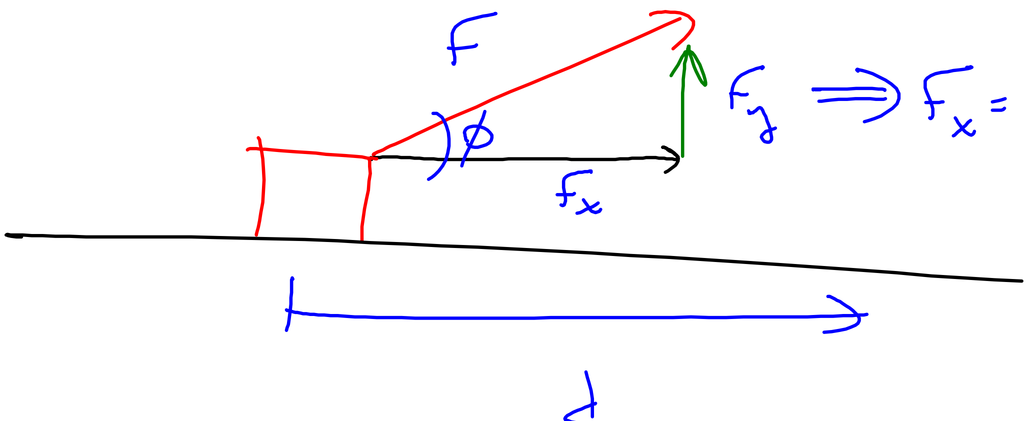


FIG. 7-32 Problem 21.

Since it's moving up the ramp at constant v, $\Delta K = 0$

thus, the work done by $\vec{F}_a$ is equal to the negative work done by the force of gravity $\Rightarrow W_a = -W_g$

$$W_a = -W_g = --F_g h = +mgh$$
$$= (3\,kg)(9.8\,m/s^2)(0.15\,m)$$
$$= \boxed{4.41\,J}$$



$\Rightarrow F_x = F\cos\phi$

$$W = F \cdot d \Rightarrow F\cos\phi \cdot d$$
$$= F \cdot d\cos\phi$$

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ΔK & bullets

an example of energy transfer

Video

1) 0.45 ACP
2) Hollow point vs. ballistic gel
3) Hollow point vs. full metal jacket
4) 9mm FMJ vs HP

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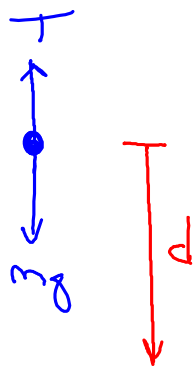
ex An elevator cab of mass $m = 500\,kg$ is descending with speed $v_i = 4\,m/s$ when its supporting cable begins to slip, allowing it to fall with constant acceleration $a = \frac{g}{5}$.

(a) During a fall through a distance $d = 12\,m$, what is the work W_g done on the cab by the gravitational force?

☆ FBD



$$W_g = F d \cos\phi$$

$$= m g d \cos\phi$$

$$= (500\,kg)(9.8\,m/s^2)(12\,m)\underbrace{\cos 0°}_{+1}$$

$$= +58{,}800\,J$$

$$= \boxed{58.8\,kJ}$$

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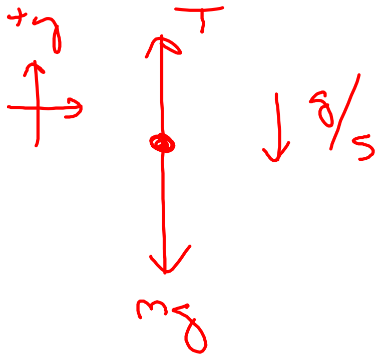
(b) During the 12m fall, what is the work W_T done on the cab by the upward pull T of the cable?



$$\underbrace{\sum F}_{m\left(-\frac{g}{5}\right)} = T - mg$$

$$m\left(-\frac{g}{5}\right) = T - mg$$

$$T = 1mg - \frac{1}{5}mg = \frac{4}{5}mg$$

$$W_T = F d \cos\phi$$

$$= T d \cos\phi$$

$$= \frac{4}{5} m g d \cos\phi$$

$$= \frac{4}{5}(500\,\text{kg})(9.8\,\text{m/s}^2)(12\,\text{m})\left(\overset{-1}{\cancel{\cos 180^\circ}}\right)$$

$$= -47{,}040\ \text{J}$$

$$= -47.04\ \text{kJ}$$

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(c) What is the net work done on the cab during the fall?

$$W_g + W_T = +59\text{ kJ} + {}^{-}47\text{ kJ} = \boxed{+12\text{ kJ}}$$

positive

WHY?

think $W = \Delta K$

(d) What is the cab's kinetic energy at the end of the 12 m fall?

$$W = \Delta K = K_f - K_i$$

$$W = K_f - K_i \Rightarrow K_f = W + K_i$$

4,000 J

$$= +12{,}000\text{ J} + \tfrac{1}{2}(500\text{ kg})(4\text{ m/s})^2$$

$$= 16{,}000\text{ J} = \boxed{16\text{ kJ}}$$

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⇒ Alternatively

$v_i = 4\,m/s$

$a = 8/5$

$d = 12\,m$

$v_f = ?$

$$a = \frac{v^2 - v_0^2}{2x} \Rightarrow v = \sqrt{2ax + v_0^2}$$

$$= \sqrt{2(1.96\,m/s^2)(12\,m) + (4\,m/s)^2}$$

$$= 7.940\,m/s$$

$$K_i = \tfrac{1}{2}(500\,kg)(4\,m/s)^2 = 4{,}000\,J = 4\,kJ$$

$$K_f = \tfrac{1}{2}(500\,kg)(7.940\,m/s)^2 = 15{,}760\,J$$

$$\approx 16{,}000\,J = \boxed{16\,kJ}$$

it's the same!

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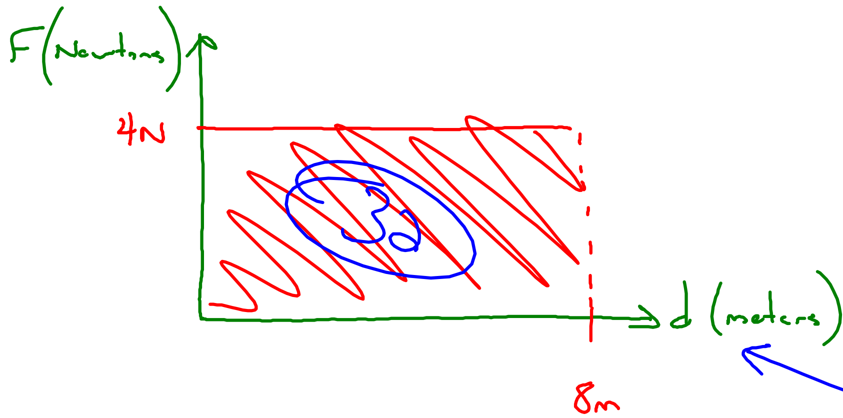
Work done by a variable force

first, let's look at the case with constant F "$W = F \cdot d$"



distance...
NOT time!

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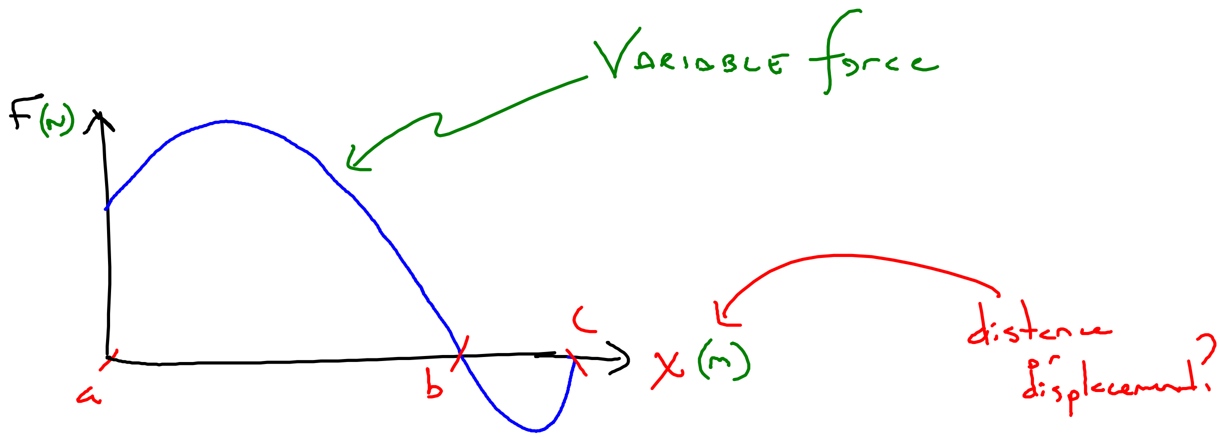
now, let's look at a variable force

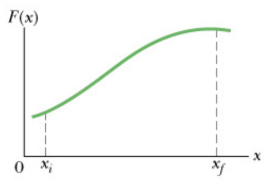


Work from $x = a$ to $x = b$ is given by $\int_{x=a}^{x=b} F(x)\, dx$

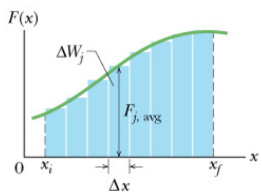
How about $x = b$ to $x = c$? $\int_{x=b}^{x=c} F(x)\, dx$

How about $x = a$ to $x = c$ (total)?

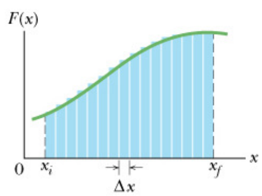
$$\int_a^c F(x)\,dx$$



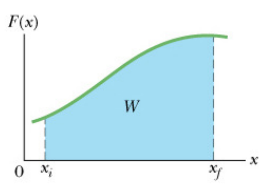
(a)



(b)



(c)



(d)

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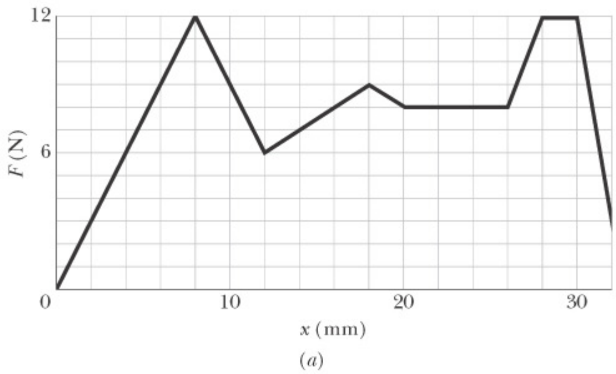
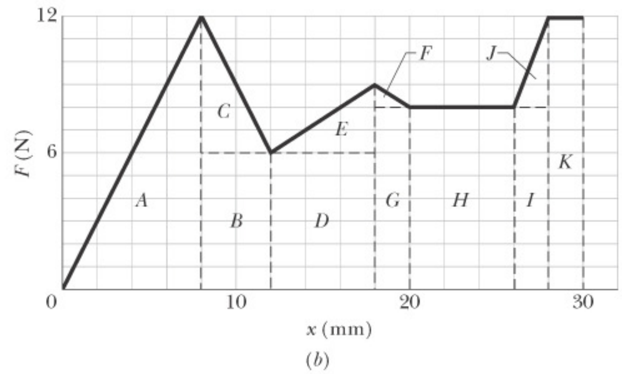
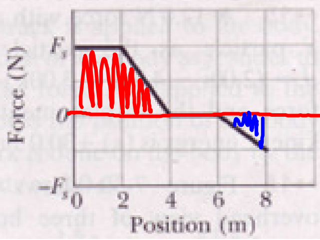




* also, given an acceleration graph ⇒ WHAT DO YOU DO?

$\Rightarrow$ Examples using $W = \int_{x_i}^{x_f} F(x)\,dx$

•34 A 5.0 kg block moves in a straight line on a horizontal frictionless surface under the influence of a force that varies with position as shown in Fig. 7-39. The scale of the figure's vertical axis is set by F_s = 10.0 N. How much work is done by the force as the block moves from the origin to x = 8.0 m?



[redacted]

200N

•36 A 10 kg brick moves along an x axis. Its acceleration as a function of its position is shown in Fig. 7-40. The scale of the figure's vertical axis is set by a_s = 20.0 m/s^2. What is the net work performed on the brick by the force causing the acceleration as the brick moves from x = 0 to x = 8.0 m? ILW

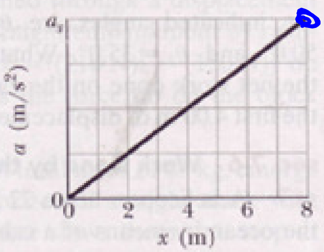


FIG. 7-40 Problem 36.

••40 A 1.5 kg block is initially at rest on a horizontal frictionless surface when a horizontal force along an x axis is applied to the block. The force is given by $\vec{F}(x) = (2.5 - x^2)\hat{\text{i}}$ N, where x is in meters and the initial position of the block is x = 0. (a) What is the kinetic energy of the block as it passes through x = 2.0 m? (b) What is the maximum kinetic energy of the block between x = 0 and x = 2.0 m?

$W = \Delta K$

$W = K_f - \cancel{K_i}^{\,0}$

$W = \int_0^2 (2.5 - x^2)\,dx$

⇒ ex from an equation (can't use $\int_{x_i}^{x_f} F(x)\,dx$) WHY?

$m = 8\,kg$, from $t = 1s$ to $t = 5s$

$x = 4t^4 - 5t^2 + 6t + 3$

$v = 16t^3 - 10t + 6$

$a = 48t^2 - 10$

$$\cancel{\int_1^5 (8kg)(48t^2 - 10)\,dx}$$

$v_i =$

$v_f =$

$W = \Delta K = K_f - K_i$

$= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

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⇒ Be careful!

Could we integrate $\int F(x)\,dx$

if $a = 48t^2 - 10$ and $F = ma$?

How about... $F = e^{-4x^2}$?

$$W = \int_{0\,m}^{3\,m} e^{-4x^2}\,dx$$

$$W = \Delta K = K_f - \cancelto{0}{K_i}$$

$$\int_0^3 e^{-4x^2}\,dx = \frac{1}{2} m v_f^2$$

assign hw

☆ Homework Assignment ☆

- HRW Chapter 7 Questions 7 & 8
  Problems 17, 34, 36, 37, 40

☆ Due Monday ☆

- Read & take notes HRW 8-1, 8-2, 8-3, 8-4
  ☆ Due Tuesday ☆

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